DESIGN AND CHARACTERIZATION OF PERMANENT MAGNETIC SOLENOIDS FOR REGAE
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Abstract
REGAE is a small electron linear accelerator at DESY. In order to focus short and low charged electron bunches down to a few μm permanent magnetic solenoids were designed, assembled and field measurements were done. Due to a shortage of space close to the operation area an in-vacuum solution has been chosen. Furthermore a two-ring design made of wedges has been preferred in terms of beam dynamic issues. To keep the field quality of a piecewise built magnet still high a sorting algorithm for the wedge arrangement has been developed and used for the construction of the magnets. The magnetic field of these solenoids has been measured with high precision and has been compared to the simulated magnetic field.

INTRODUCTION
The Relativistic Electron Gun for Atomic Exploration (REGAE) is a small 5 MeV linear accelerator at DESY in Hamburg, which produces short, low emittance electron bunches. It originally was meant for temporal resolving electron diffraction experiments [1]. But two further experiments are currently planned at REGAE. First, an external injection experiment for Laser Wakefield Acceleration (LWA) [2] will be performed in the framework of the LAOLA collaboration (LAboratory fOr Laser- and beam-driven plasma Acceleration). This experiment will provide a method for the reconstruction of the electric field distribution within a linear plasma wakefield. Second one is an extension of the original experiment. A time resolving high energy Transmission Electron Microscope (TEM) will be set up.

Both experiments require strong focusing magnets inside the new target chamber at REGAE. Permanent magnetic solenoids (PMSs) can provide the needed focusing strength due to their enormous surface current density, while having compact dimensions at the same time. Since short and strong solenoids, as required for REGAE, exhibit a distinct non-linearity, the induced emittance growth is relatively large and has to be minimized as far as possible. Furthermore, the focusing strength is not adjustable and 3D in-vacuum movers are required for positioning the magnets. Due to the chosen movers a weight limitation for the magnets reveals as an additional requirement. Overcoming these difficulties PMSs are an interesting alternative when a low energy beam has to be strongly focused.

DESIGN
A strong focusing is needed to generate a small transverse beam size for the external injection experiment and for a large magnification in the transmission electron microscope.

FIELD DESCRIPTION AND SORTING ALGORITHM
Field Model
Due to the necessity of a model which describes the resulting field of 24 wedges we developed a simple model: each wedge is described by current loops covering the surface (Fig. 2 b), where each loop can be divided into four straight parts. The magnetic field can be calculated by means of Biot-Savart's law for a straight wire. The magnetization M of a wedge is defined by the direction and its magnitude which were measured by the manufacturer and can be translated into a tilt of the current loops or a variation of the current,
respectively. The average current is chosen in a way that it reproduces the measured maximum longitudinal magnetic field. The manufacturing errors are added to this average current proportionally.

In order to improve the performance of the calculation routine it is possible to reduce the number of current loops \( N \) per wedge but in contrast gain some field uncertainties. In Fig. 2 a) these uncertainties are illustrated as the deviation from the converged 2nd field integral \( (N \to \infty) \), which is proportional to the focus strength of a solenoid

\[
\int_{-\infty}^{\infty} B_{z,0}^2(z) \, dz.
\]

Here \( B_{z,0} \) is the longitudinal on-axis magnetic field component. We decided to use \( N = 20 \) current loops per wedge and end up with a field integral accuracy of \( \sim 1\% \).

![Figure 2: a) Relative deviation of the 2nd field integral for different number of current loops. b) Model of a wedge with current loops and a tilted magnetization.](image)

**Sorting Algorithm**

Since the total magnetic field of the two-ring setup is simply given as the superposition of the individual magnetic fields of the wedges (in theory) an optimal configuration of the 24 wedges can be found. The optimal configuration could for example be a case where errors in the direction of the magnetization vector \( \mathbf{M}_{ij} \) (\( i \) denotes ring 1 or 2 and \( j \in [1,12] \)) compensate each other. In order to quantify the definition of the optimal configuration the fit criterion

\[
\sum_{k=0}^{N} (I(r, \theta_k)p - I(r, \theta_k)np)^2 \tag{1}
\]

with

\[
I(r, \theta_k) = \int_{z_{min}}^{z_{max}} B_r(r, z) \, dz \bigg|_{\theta_k}
\]

was used, where \( B_r(r, z) \) is the radial magnetic field at distance \( r \) from the geometrical z-axis, \( \theta_k \) the \( k \)th rotation angle around the z-axis, \( p \) and \( np \) denote a solenoid with flawless and flawed wedges, respectively. This *fitness value* has to be minimized in order to achieve maximum field symmetry and to minimize higher order (quadrupole, sextupole, etc.) field components, which might have detrimental effect on beam dynamics.

Using the computationally simple field description shown above an algorithm has been developed with the goal to find the optimal permutation for each ring given the measured magnetization data provided by the manufacturer. This was necessary because the calculation time by a brute force computation would not be feasible (1 ring \( \to 12! \approx 5 \times 10^8 \) permutations). The algorithm is a two-step process. The first step consists of a numerical least-square algorithm, which determines a rough starting point for the second part. Here all available wedges - including spare wedges - are taken into account. In each iteration three actions can be performed:

- Swap with pool (incl. spares)
- Swap inside the rings
- Flip around radial axis.

The second step is based on the concept of *simulated annealing* [4]. Simulated annealing tries to find the global minimum of a fitness function like Eq. 1 by treating the system as a thermodynamical system with falling temperature \( T \). For each iteration the fitness function \( f(x) \) is determined. Also \( T \) is lowered according to a predefined sequence. In our case each iteration consists of swapping wedges inside the rings or flipping them around the radial axis. \( x \) corresponds to a certain permutation of both rings whereas \( x_{opt} \) is the current best solution. If \( f(x) \leq f(x_{opt}) \), \( x_{opt} = x \). If \( f(x) > f(x_{opt}) \), \( x_{opt} = x \) only with a probability

\[
\exp\left(-\frac{f(x_{opt}) - f(x)}{T}\right). \tag{2}
\]

From Eq. 2 it can be seen that for low temperatures \( T \) the probability of choosing the permutation decreases, whereas for high \( T \) the algorithm tends to *jump* out of minima more often. This helps to avoid trapping in local minima.

**Results**

The permutations provided by the algorithm were then used to determine emittance growth. To this end full 3D field maps were calculated for both the flawed and flawless wedge case using the analytical field model. These were then used for particle tracking using ASTRA [5]. Because the emittance growth depends on the initial beam parameters the results are only valid and comparable for the chosen parameters as well. We used a 5 MeV beam with an RMS beam size of 600 \( \mu \)m. Two assemblies were found with a relative emittance growth \( \epsilon_{np}/\epsilon_p \) of 1.04 and 1.15, respectively. An arbitrary assembly reaches an average relative emittance growth of \( \sim 1.30 \).

**FIELD MEASUREMENT**

**3D-Hall Probe**

Measuring the magnetic field of a geometrical small magnet with a high precision is challenging. In order to com-
pare the field simulations with a measurement we decided for the Metrolab Three-axis Hall Magnetometer THM1176-
HF [6] which provides the required accuracy. With a sensor
housing of 5.1 mm × 1.3 mm the geometrical dimensions of
the probe are small enough to measure the magnetic field
inside the PMS. Furthermore its small active volume of
(150 × 150 × 10) μm³ is sufficient to measure the absolute
field despite the high field gradients.
The Hall probe was calibrated relatively to an NMR Teslame-
ter. The absolute as well as the relative accuracy meet our
requirements of 10⁻⁴. From a linear regression of the absolu-
te accuracy measurement follows for the slope 0.999 98(9)
and for the offset 1.08(3) × 10⁻⁴. The number inside the
brackets denotes the uncertainty of the last digit.

**Magnetic Field Measurement and Post-Processing**

The field measurements were done with a 3D linear stage
with a minimal step size of 12.5 μm. The PMS was fixed on
a triple-axis adjustment table. The solenoidal field (Fig. 3)
itself offers the possibility to align the linear stage relatively
to the magnetic field of the PMS. In its transverse plane
at the position of the maximum longitudinal field it has only
a longitudinal field component. This fact can be used to
align the horizontal and vertical axes of the Hall probe. At
the longitudinal position of the zero-crossing of the mag-
etic field all field components are equal to zero. These two
points define the symmetry axis of the solenoid which can
be used to align the PMS with respect to the longitudinal
axes of the linear stage. Because already small deviations
of the alignment can cause big deviations of the measured
from the simulated magnetic field a post-processing align-
ment is necessary to compare simulation and measurement.
There are 9 degrees of freedom for the alignment: 3 trans-
slational (x, y, z) and 3 rotational degrees (β, β, β) of the
PMS with respect to the linear stage and 3 rotational degrees
(α, α, α) of the Hall probe itself. The indices denote
the rotation axes and (x, y, z) the horizontal, vertical and
longitudinal axes, respectively. The aforementioned field
simulation code has been extended by 3 rotational degrees
for the PMS geometry. As described, the other two rotational
degrees of the probe are included by the field measurement
itself. The calculated values are α = -7.41(4) × 10⁻³ rad
and α = 4.43(4) × 10⁻³ rad. Because the simulated field is
a rectangular array the 3 translational degrees are introduced
as a shift of the simulated field with respect to the measured
one. In order to do this the resolution of the simulated field
has to be higher. As a consequence, the translation align-
ment is discrete and not continuous like the rotational align-
ment. The rotation around the longitudinal axis of the probe α is
introduced as a simple rotation of the field vector around the
z-axis.
For the fitting routine a simple least-square criterion is chosen:

\[ \chi^2 = \sum_i^N \frac{(B_{m,i} - B_{s,i})^2}{\sigma_B} \]

where \( B_{m,i} \) is the measured magnetic field vector, \( B_{s,i} \) is the
simulated magnetic field vector, \( \sigma_B \) is the standard deviation
of the repeatedly measured field and \( N \) is the sample size. In
order to proof the goodness of the fit the reduced Chi Square
\( \tilde{\chi}^2 = \chi^2/(N - n - 1) \) is introduced which should be close
to 1. For one of the PMS the field was measured and compared
to both assembled PMS models as well as the ideal PMS model
with flawless wedges. For the fit 27 magnetic field vectors
around each aforementioned zero-crossings were taken into
account. The step size of the grid was (50 × 50 × 50) μm³.
The results of the fits are shown in Tab. 1. The resolution of
the simulated grid was (16.7 × 16.7 × 12.5) μm³ and hence
the minimal translation step widths of the fit routine.

<table>
<thead>
<tr>
<th>PMS 1</th>
<th>PMS 2</th>
<th>Ideal</th>
</tr>
</thead>
<tbody>
<tr>
<td>x [mm]</td>
<td>0.0333</td>
<td>-0.0333</td>
</tr>
<tr>
<td>y [mm]</td>
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<td>0</td>
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<tr>
<td>z [mm]</td>
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<td>-0.025</td>
</tr>
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<td>βx [rad]</td>
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</tr>
<tr>
<td>βy [rad]</td>
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<tr>
<td>βz [rad]</td>
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</tr>
<tr>
<td>αx [rad]</td>
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<td>0.024</td>
</tr>
<tr>
<td>αy [rad]</td>
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<td></td>
</tr>
<tr>
<td>αz [rad]</td>
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<td></td>
</tr>
<tr>
<td>( \tilde{\chi}^2 )</td>
<td>0.439</td>
<td>0.460</td>
</tr>
</tbody>
</table>

Figure 3: Measured (blue dots) and simulated (red dashed)
on-axis magnetic field \( B_z \).

**CONCLUSION**

The goodness of the fit indicates an overestimation of the
measurement errors. Furthermore the results for the two
different PMS are almost the same which indicates that it
is not possible to resolve the smallest deviations of the two
PMS. Nevertheless we were able to develop an analytical and
fast magnetic field simulation tool which is very general and
not limited to solenoidal fields. Furthermore the comparison
between the measurement and the simulation was successful
despite the challenging magnetic field measurement.
REFERENCES


