SURFACE RESISTANCE RF MEASUREMENTS OF MATERIALS USED FOR ACCELERATOR VACUUM CHAMBERS

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Abstract

The RF surface resistance of accelerator vacuum chamber walls can have a significant impact on the beam quality. There is a need to know how the use of a new material, surface coating or surface treatment can affect the RF surface resistance. ASTeC and Lancaster University have designed and built two test cavities where one face can be replaced with a sample in the form of a flat plate. The measurements are performed with a network analyser at the resonant frequency of approximately 7.8 GHz.

INTRODUCTION

If one considers the formulation of the unloaded quality factor $Q_0$ of an RF cavity [1] one can write

$$Q_0 = \frac{2\pi f_0 \mu_0 \int S |H|^2 \, dV}{\int S R_S |H|^2 \, dS}$$

where $H$ is the magnetic field, $R_S$ is the surface resistance of the cavity walls and $f_0$ is the resonant angular frequency of the cavity. To accommodate the possibility of a cavity being comprised of two parts (a cavity and a sample) which could be made of different metals or otherwise have different $R_S$ values, one can most conveniently rewrite this as

$$Q_0 = \frac{G}{R_S^{\text{sample}} + R_S^{\text{cavity}}}$$

where $G$ is the geometry constant of the cavity [1], defined as

$$G = \frac{2\pi f_0 \mu_0 \int S |H|^2 \, dV}{\int S |H|^2 \, dS}$$

$R_S^{\text{sample}}$ and $R_S^{\text{cavity}}$ are the surface resistance of the sample and the cavity respectively, and $p_S$ and $p_C$ the sample and cavity ratios – the proportion of the total field dissipated over their respective surfaces, i.e.

$$p_S = \frac{\int S^{\text{sample}} |H|^2 \, dS}{\int S |H|^2 \, dS}$$

$$p_C = \frac{\int S^{\text{cavity}} |H|^2 \, dS}{\int S |H|^2 \, dS} = 1 - p_S$$

For any similarly-shaped cavity $G$ and $p_S$ are in principle constant, irrespective of the materials used.

This implies that, knowing $R_S^{\text{cavity}}$, $G$ and $p_S$ for a given cavity we can calculate $R_S$ for any sample by placing it on top of the cavity and finding the unloaded $Q$-factor of the resulting RF resonance.

$$R_S^{\text{Sample}} = \frac{G/Q_0 - R_S^{\text{cavity}} (1-p_S)}{p_S}$$

METHOD

Calculation of $Q_0$

Two double-choked pillbox-type cavities were used to take our measurements, one of which can be seen in Fig. 1. The choked cavity allows the testing of flat samples without the need for flanges and RF seals. Both cavities were manufactured to identical dimensions by Niowave Inc. [2], one being made from aluminium and one from niobium.

![Figure 1: A two-choked 8 GHz Al test cavity.](image)

In each case the samples, in the form of flat plates or discs of sufficient width to completely cover the outer choke, were placed on top of the cavity with spacers providing a gap of ~2 mm between the cavity and the sample. An axially-mounted coaxial antenna was attached to a calibrated network analyser to induce RF resonance, and the coefficient of signal reflection ($S_{11}$) measured against frequency. Initial setup required that the spacing between the sample and the cavity was adjusted to
maximise signal loss near the resonant frequency of the first mode (approximately 7.8 GHz). The probe depth was adjusted to induce near-critical coupling, as judged from a Smith Chart of $S_{11}$. [3]

$Q_0$ was calculated using the formula

$$Q_0 = \frac{f_0}{f_2 + f_3 - f_1}$$  \hspace{1cm} (7)

where $f_1$ and $f_2$ are the frequencies at which the imaginary components of $S_{11}$ are minimal and maximal respectively with the system in the detuned open position. $f_3$ and $f_4$ are the frequencies at which the imaginary component of $S_{11}$ are ±1 respectively in the detuned open position. [3]

**Calculation of Surface Resistance from First Principles**

The surface resistance $R_S$ of a metal under AC stimulation depends on four factors; its bulk electrical resistivity $\rho$ and magnetic permeability $\mu$, the AC frequency $f$ and its surface roughness. In the GHz regime all four are important contributors to $R_S$. For a perfectly smooth metal surface (with zero roughness)

$$R_S = \sqrt{\pi\mu_0 f \rho}$$  \hspace{1cm} (8)

to account for the effect of the finite skin depth in the metals under AC excitation [1].

Hammerstad and Bekkadal (1975) produced an empirical formula describing the effect of the RMS roughness, $R_Q$, on $R_S$. Based on their observations [4] an additional factor applies as follows:

$$R_S = \left(1 + \frac{2}{\pi} \tan^{-1} \left(1.4 \times R_Q^2 \times \frac{\mu f}{\rho}\right) \right)$$  \hspace{1cm} (9)

The sample surface roughness was calculated using measurement data from an interferometric microscope by scanning the surfaces of five metal samples: metal discs made of Cu, Al, Nb and 304 Stainless Steel and a ~5 µm-thick Cu film deposited via pulsed DC magnetron sputtering onto a Silicon (100) wafer.

A theoretical value of $R_S$ was then calculated for each sample using the modified formula (9) above.

Due to its physical dimensions the available interferometric microscope could not be used to obtain a roughness profile for the surface of the cavities themselves. As a consequence, only an upper limit was set on their $R_Q$, and hence $R_S$, based on the manufacturer’s specifications.

**Comparison of Measured and Theoretical Results**

The first step in an attempt to validate this method was to plot the calculated and measured values of $R_S^{\text{sample}}$ for all cavities against one another. The data from both cavities was observed to be in good agreement (coefficient of determination > 0.97) to a linear relationship. A manual iterative method was used to find the values of $R_S^{\text{cavity}}$, G and $p_S$ for which the relationship most closely approximated $y = x$. As would be expected such values of $G$ and $p_S$ were the same for both cavities, at ~224 and ~0.37 respectively.

These figures were then used as the starting point for a more precise fitting technique, using MathCAD [5]. Here, for each value of $p_S$ and $R_S^{\text{cavity}}$, $R_S^{\text{sample}}$ was swept across a small range of values and the point at which both cavities returned the same value of $G$ was logged. It was observed that the returned value of $G$ was 225 for all sample-cavity combinations, to within the standard deviation of the measurements, when $p_S = 0.375$.

This matched very closely with values for $G$ and $p_S$ calculated from first principles using a CST [6] Microwave Studio simulation (shown in Fig. 2): $G = 224$ and $p_S = 0.375$.

![Figure 2: Simulated distribution of the H-field on the sample (top) and cavity and chokes (bottom).](image)

**RESULTS AND DISCUSSION**

Table 1 shows the calculated values of $R_S$ at RF frequency $f = 7.8$ GHz.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\rho$(Ωm)</th>
<th>$R_Q$(m)</th>
<th>$R_S$(mΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu plate</td>
<td>1.72×10^{-8} [7]</td>
<td>4.09×10^{-7}</td>
<td>28.6</td>
</tr>
<tr>
<td>Al</td>
<td>2.73×10^{-8} [7]</td>
<td>4.05×10^{-7}</td>
<td>34.0</td>
</tr>
<tr>
<td>304 SS</td>
<td>7.20×10^{-7} [8]</td>
<td>1.44×10^{-6}</td>
<td>160</td>
</tr>
<tr>
<td>Nb</td>
<td>1.52×10^{-7} [7]</td>
<td>(1×10^{-6})</td>
<td>80.7</td>
</tr>
<tr>
<td>Cu film</td>
<td>1.72×10^{-8} [7]</td>
<td>9.08×10^{-6}</td>
<td>22.7</td>
</tr>
</tbody>
</table>

Note that $\mu \approx \mu_0$ [7, 8] for all the materials we used. Table 2 shows the mean value of $Q_0$ for each cavity-sample combination from sets of five consecutive calculations - removing, rotating and replacing the sample between each one.

The uncertainty comes from combining (as the root of the sum of the squares) the relative standard deviation within these sets of readings and the estimated relative error in the measurements of $f_0$, $f_1$, $f_2$, $f_3$ and $f_4$. 

Table 2 shows the mean value of $Q_0$ for each cavity-sample combination from sets of five consecutive calculations - removing, rotating and replacing the sample between each one.
Table 3 shows the resultant values of $R_S^{sample}$ for each cavity-sample combination, as well as those calculated from first principles.

The calculations used some values which it was not possible to obtain from literature or determine from direct measurement:

- For both cavities a value of $G = 255$ and $p_S = 0.375$ were used, from the MathCAD best-fit solution (supported by the CST calculations).
- $Q_0$ for the cavities was assumed to be that which gave the best fit to the data.
- $Q_0$ for the Nb plate comes from the manufacturer’s specifications.

Table 3: Comparison of the Values of $R_S$ calculated from First Principles and from the $Q_0$ Readings for 7.8 GHz Al and Nb Cavities

<table>
<thead>
<tr>
<th>Sample</th>
<th>$R_S$ calculated (Ω)</th>
<th>$R_S^{sample}$ from $Q_0$, Al (Ω)</th>
<th>$R_S^{sample}$ from $Q_0$, Nb (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu film</td>
<td>$2.27 \times 10^{-2}$</td>
<td>$2.84 \times 10^{-2}$</td>
<td>$2.34 \times 10^{-2}$</td>
</tr>
<tr>
<td>Cu plate</td>
<td>$2.86 \times 10^{-2}$</td>
<td>$2.70 \times 10^{-2}$</td>
<td>$2.09 \times 10^{-2}$</td>
</tr>
<tr>
<td>Al</td>
<td>$3.36 \times 10^{-2}$</td>
<td>$3.85\times 10^{-2}$</td>
<td>$4.43 \times 10^{-2}$</td>
</tr>
<tr>
<td>304 SS</td>
<td>$1.60 \times 10^{-1}$</td>
<td>$1.68\times 10^{-1}$</td>
<td>$1.52 \times 10^{-1}$</td>
</tr>
<tr>
<td>Nb</td>
<td>$8.06 \times 10^{-2}$</td>
<td>$6.75\times 10^{-2}$</td>
<td>$6.49 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

The results suggest that this is have here a useful and robust method for determining $R_S^{sample}$. The internal consistency of our results suggests that its effect on $Q_0$ is as is expected, and that $G$, $p_S$ and $p_C$ can be accurately calculated for a cavity of this sort using CST Microwave Studio. The empirical formula for the surface resistance of a rough surface means that we can either calculate $R_S^{cavity}$ from first principles or, if measuring the cavity $Q_0$ is not practical, find a good estimate for it via the best fit to the data from several ‘calibration’ samples. Therefore, once we measure $Q_0$ on that cavity for each subsequent unknown sample we have all the components we need to calculate $R_S^{sample}$.

Possible sources of systematic error include:

- The assumption that the metal remains in the normal skin-depth regime.
- The roughness-modified formula for $R_S$ is only an approximation.
- The fact that the samples we used might have a different bulk resistivity to that given by the literature.
- Surface oxidation, dirt, and/or fractures beneath the surface of the sample could all also have had an effect on $R_S$ which is not currently quantifiable.
- Coupling losses cannot be accounted for.
- The cavity was originally designed to measure $R_S^{sample}$ at cryogenic temperatures [9]. If the bandwidth permits, we will try to duplicate the measurements using the method described above, but we plan to use calorimetric methods which will afford a far more reliable method of measuring the much-higher $Q$-factors. Additional considerations, and details of the apparatus, are covered in another paper [9].

CONCLUSION

The method of measuring RF surface resistance using two-choke test cavities at room temperature was analytically developed and implemented in two cavities made of Al and Nb. Measured values of $R_S$ for Cu, Al, Nb and 304 stainless steel are in a good agreement with theoretically calculated values.

REFERENCES

[9] P. Goudket et al., Test Cavity for SRF Thin Film Evaluation, these proceedings.