ANALYSES OF LIGHT'S ORBITAL ANGULAR MOMENTUM FROM HELICAL UNDULATOR HARMONICS*

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Abstract
Spiral interference patterns between two different harmonic radiations from two tandem-aligned helical undulators were observed by using a scanning fiber multi-channel spectrometer and a UV-CCD camera placed at the beamline of S1 straight section in UVSOR-III storage ring. By a series of measurements, various interference patterns such as single, double, and triple spirals were observed which concur with the theoretical predictions for every mode in the right or left circular polarization. The rotation of an interference pattern by rotating a polarizer was also observed.

INTRODUCTION
A photon beam propagating in the vacuum may carry a quantized orbital angular momentum (OAM) [1, 2]. Based on this fact, many applications in the visible wavelength regime to use light’s OAM has been reported such as the manipulation of small particles [3-5], quantum entanglement [6], vector vortex coronagraph for astronomy [7], efficient mode conversion for OAM resolved spectroscopy [8], and so on. On the other hand in the shorter wavelength regime, there have been not many activities on this exotic property due to some difficulties to endow and control such a property. However, after the theoretical prediction that higher harmonic radiation from a helical undulator carries OAM was made [9, 10], this novel property attracts a great deal of attention because it may be used as a new probe for synchrotron radiation science that would be performed in a diffraction limited light source facility such as the MAX-IV, NSLS-II, or APS-II. Although the diffraction limited x-ray source does not yet exist, the first experimental evidence that the second harmonic radiation from a helical undulator carries OAM was presented by Bahrdt, et al. at BESSY II [11]. Successive systematic experiments have been done at the UVSOR-III. Here we present experimental results and analyses on OAM properties of undulator harmonic radiation.

EXPERIMENTAL SETUP
The 750 MeV UVSOR-III is already a diffraction limited light source in the UV region. In this ring, a tandem-aligned double-APPLE undulator system similar to that in BESSY II is installed for FEL and coherent light source experiments. Using this set-up with a few reduced ring energies, we observed spiral interference patterns between two different harmonic radiations with a scanning fiber multi-channel spectrometer and a CCD camera placed at the end of BL1U Beamline. By these measurements, various interference patterns such as single, double, and triple spirals were observed which concur with the theoretical prediction for every mode in the right or left circular polarization. The rotation of an interference pattern by rotating a polarizer was also observed.

Table 1 shows the total emittance and corresponding diffraction limited wave length at each stored electron energy.

<table>
<thead>
<tr>
<th>Ring Energy</th>
<th>Emittance</th>
<th>Diffraction Limit. Wave Length</th>
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<tbody>
<tr>
<td>750 MeV</td>
<td>17.5 nm-rad</td>
<td>220 nm (6 eV)</td>
</tr>
<tr>
<td>600 MeV</td>
<td>10.9 nm-rad</td>
<td>138 nm (9 eV)</td>
</tr>
<tr>
<td>500 MeV</td>
<td>7.6 nm-rad</td>
<td>100 nm (12 eV)</td>
</tr>
<tr>
<td>400 MeV</td>
<td>4.8 nm-rad</td>
<td>63 nm (20 eV)</td>
</tr>
</tbody>
</table>

The top view of undulator straight section of UVSOR-III ring is shown in Fig. 1.

Figure 1: Top view of undulator straight section used for experiment.

Figure 2 shows a schematic view of experimental setup for the intensity distribution measurement with a...
spectrometer on an x-y stage. Scanning was performed in the areal range of 20mm x 20mm. Scanning was performed in the areal range of 20mm x 20mm.

Figure 2: Schematic view of experimental setup. The distance from the downstream undulator to the spectrometer is 6.9 m, and the separation between two undulators (center-to-center) is 1.6 m.

Figure 3 shows examples of measured spiral patterns for interference between the circularly polarized second harmonic and the linearly polarized fundamental radiation. The measurement was done at the photon wavelength of 245 nm. Dotted spiral line represents the calculated pattern with the equation in ref. 11 for the additive interference between two undulator radiations. The direction of spiral rotation depends on the helicity of circular polarization. In order to observe clear interference pattern, the stored electron energy was reduced from the nominal 750 MeV to 600 MeV though the emittance of nominal user operation energy is already the diffraction limited emittance for $\lambda=245$ nm.

The double spiral patterns by the fundamental and the third harmonic interference are shown in Fig. 4. The measurement was done at the photon wavelength of 248 nm. Dotted spiral line represents the calculated pattern with the extended equation based on the equation in ref. 11 for the additive interference between two undulator radiations. The direction of spiral rotation depends on the helicity of circular polarization. In these measurements, the electron energy was reduced down to 500 MeV. These images were taken by scanning a fiberscope spectrometer.

Figure 3: Interference pattern observed by scanning the fiberscope spectrometer. Upstream U: circular 2nd harmonic, Downstream U: linear fundamental. E=600 MeV, $\varepsilon_0=11$ nm-rad, $\lambda=245$ nm. Dotted spiral line represents the calculated pattern with the equation in ref. 11.

The following interference measurements were performed using a band-pass filter and a CCD camera. Figure 5 shows a schematic layout of experiment. A laser-quality band-pass filter is inserted in front of a CCD camera, and a polarizer is also inserted upon necessity.

An example of observed pattern is shown in Fig. 6. This figure demonstrates the rotation of spiral pattern by rotating a polarimeter. This rotating spiral pattern was observed for the interference between the circular fundamental (upstream, right-handed polarization) and the second harmonic (downstream, left-handed polarization with $\ell=-1$) with the rotation of polarimeter.

For this measurement, the electron energy was 600 MeV, and the emittance was 11nm-rad. The wavelength used for observation was 240 nm. The rotation angles of polarimeter are: Top Left: 0°, Top Right: 30°, Bottom Left: 60°, Bottom Right: 90°, respectively. The rotation angle of spiral pattern is double degree and the rotation direction is opposite to that of polarimeter.

Similar measurements for the case of double and triple spiral interference were made. In the double spiral case, the rotation angle of spiral pattern was found to be the same angle but the opposite direction with the polarimeter rotation.
Figure 4: Interference pattern observed by scanning the fiberscope spectrometer. Upstream U: linear 1\textsuperscript{st} harmonic, Downstream U: circular 3\textsuperscript{rd} harmonic. $E=500$ MeV, $\epsilon_0=8$ nm-rad, $\lambda=248$ nm. Dotted spiral line represents the calculated pattern with the extended equation based on that in ref. 11.

Figure 5: Schematic view of experimental setup. A laser-quality band-pass filter is inserted in front of CCD camera.

ANALYSES

As we see in the previous section, the spiral pattern appears only with inserting a polarimeter in the case of interference between upstream source and downstream source having opposite helicities, and the direction of spiral rotation is opposite to that of polarimeter rotation. The rotation angle of interference pattern was found to be $2/|\Delta L|$ radian in the opposite direction to polarimeter rotation for every combination of different quantum numbers, i.e. a double angle for $\Delta L=1$, an equal angle for $\Delta L=2$, and $2/3$ angle for $\Delta L=3$.

CONCLUSIONS

Our systematic measurements and analyses of interference patterns for various combinations of different OAM states have revealed intrinsic properties of synchrotron radiation from a relativistic electron beam in spiral motion which concur with the theoretical prediction.

Figure 6: Spiral rotation, 1\textsuperscript{st} & 2\textsuperscript{nd} harm. interference. Rot. Angle: TL: 0°, TR: 30°, BL: 60°, BR: 90°

REFERENCES


