Abstract

Bunch length is measured at CEBAF using an invasive technique. The technique depends on applying an energy chirp for the electron bunch and imaging it through a dispersive region. Measurements are taken through Arc 1 and Arc 2 at CEBAF. The fundamental equations, procedure and recent results are given.

INTRODUCTION

CEBAF is a folded transfer line, with no natural stable bunch structure. Bunch length measurement and control are important for CEBAF because a too-long bunch would generate excessive energy spread, resulting in interrupted beam delivery due either to beam loss or failure to satisfy user requirements. Bunch length is commonly measured in the injector beam dump at up to 123 MeV using the “zero-phasing” method [1] before injection into the North Linac (NL). However, to mitigate periodic problems with control of bunch “tails,” the beam undergoes a final magnetic compression in the injection chicane (four dipoles and nine quadrupoles) linking the injector and NL. After exiting the NL at up to 1.2 GeV, the beam follows the semi-circular arc 1A to its injection point into the South Linac (SL). Arc 2A returns the beam for reinjection into the NL. This cycle may be repeated for up to six passes through the NL, after which beam is delivered to Hall D (up to 12 GeV). The first two recirculation arcs incorporate high dispersion points at the central dipoles (ηx ~ 5 meters), enabling synchrotron radiation (SR) imaging to monitor energy stability and other beam properties. Bunch length measurement generally parallels that described in [2]. We discuss the relationship between measurement of bunch length and of transverse emittance and how we have used this approach to measure the steady-state bunch length for high-power CW beam.

To minimize the energy spread for users, the beam is accelerated at peak energy gain (on-crest). By altering the global (“gang”) phase of a linac, the z-correlated energy spread may be increased. Off-crest acceleration increases the energy spread and the transverse beam size at the high dispersion monitor, enabling measurement of the bunch length. The early arcs of CEBAF are isochronous (M_{0y}=0), so the longitudinal bunch structure is preserved. Shifting the linac phase by equal amounts of opposite polarity results in turn-by-turn compensation of the added energy spread while allowing observation in the 1A synchrotron light monitor (SLM). Under these conditions, bunch length measurement is possible at high CW beam current while maintaining lossless transport to the appropriate beam dump.

SLM ANALYSIS

We begin with a coordinate system \((x,y,z,\delta)\) following the beam, where \((x,y)\) are transverse coordinates, \(z\) is the longitudinal coordinate, and \(\delta\) is the relative deviation of a particle from the average bunch energy. We simplify by ignoring transverse momenta until we discuss initial state asymmetries below. When the beam is accelerated off-crest, a \(z\)-\(p_z\) correlation (often called a chirp) is induced. Longitudinal position is re-mapped into transverse position in regions with significant dispersion. With \(E_{inj}\) as the injector energy and \(E_{NL}\) as the linac energy gain,

\[
E = E_{inj} + E_{NL} \cos(\kappa z + \phi)
\]

(1)

The linac energy gain is sinusoidal with \(k\) as the RF wave number, \(z\) as the longitudinal deviation from the bunch center at \(z=0\), and \(\phi\) is the phase shift of the accelerating RF. For short bunches and large \(\phi\), we consider \(kz < \phi\).

During off-crest acceleration, the average energy would decrease due to the cosine function. To keep the beam within the energy acceptance, the RF accelerating gradient is adjusted to hold beam energy constant.

\[
E = E_{inj} + \frac{E_{NL}}{\cos(\phi)} \cos(kz + \phi)
\]

(2)

We re-write this to first order in \(kz\) as

\[
E = E_{inj} + E_{NL} k z \tan(\phi)
\]

(3)

so that

\[
\frac{E - \bar{E}}{\bar{E}} = \frac{-E_{NL}}{E_{inj} + E_{NL}} k z \tan \phi
\]

(4)

where \(\bar{E}\) is the average beam energy and is equal to \((E_{inj} + E_{NL})\). We take the initial beam to be sufficiently symmetric that the distribution function \(F(x,y,z,\delta)\) is uncorrelated pairwise in \(x, z, \delta\). The initial mean square energy spread and bunch length may be represented as

\[
\langle z_0^2 \rangle = \int \int \int d\delta d\bar{z} dx dy \ z^2 \ F
\]

(5)

\[
\langle \delta_0^2 \rangle = \int \int \int d\delta d\bar{z} dx dy \ \delta^2 \ F
\]

(6)

Adding a linear energy chirp shears the distribution in the \(\delta\)-\(z\) plane in such a way that the altered distribution \(G\) may be written in terms of the initial function \(F\) evaluated at \(\delta = \delta - \lambda z\):

\[
G(x,y,z,\delta) = F(x,y,z,\delta - \lambda z)
\]

(7)

\[
\lambda = \frac{E_{NL} k \phi}{E_{inj} + E_{NL}}
\]

(8)
The longitudinal distribution is unchanged. The transformed mean square energy expression becomes:

\[
\langle \delta^2 \rangle = \iint \iiint d\delta dz dx dy \ \delta^2 \ G(x,y,z,\delta) \\
\langle \delta^2 \rangle = \iint \iiint d\delta dz dx dy \ (\delta + \lambda z)^2 \ F(x,y,z,\delta) \\
\langle \delta^2 \rangle = \langle \delta_0^2 \rangle + \lambda^2 \langle z_0^2 \rangle 
\]

(9)

Applying the same process to \(\langle x^2 \rangle\) at a point of high dispersion as a function of the phase offset \(\phi\), we start with

\[
\langle x^2 \rangle = \iint \iiint d\delta dz dx dy \ x^2 \ f(x,y,z,\delta) \tag{10}
\]

Consider the chirp to be applied at some non-dispersive point an integer number of betatron oscillations away. The chirp adds energy to the particle without changing its other coordinates, shifting the particle position downstream without changing the phase space density. Neglecting chromatic effects, the chirped distribution \(g(x,y,z,\delta)\) is related to the original function \(f\) by

\[
g(x,y,z,\delta) = f(\hat{x},y,z,\hat{\delta})
\]

(11)

where \(\hat{x} = x - \eta \lambda z\) and \(\hat{\delta} = \delta - \lambda z\). Transforming the expression for \(\langle x^2 \rangle\) back to the original coordinates gives

\[
\langle x^2 \rangle = \iint \iiint (x + \eta \lambda z)^2 \ f(x,y,z,\delta) \tag{12}
\]

\[
x_{rms} = \sqrt{\langle x^2 \rangle} = \sqrt{\langle \lambda_0^2 \rangle + \eta^2 \lambda^2 \langle z^2 \rangle} \tag{13}
\]

The \(\langle x^2 \rangle\) term at a dispersive location includes the effect of both emittance and initial uncorrelated energy spread. The hyperbolic relation between \(x_{rms}\) and linac phase offset is exactly parallel with the hyperbolic relation between \(x_{rms}\) and lens strength in simple quadrupole scan envelope measurements. The slope of the hyperbolic asymptotes in the quad scan is proportional to the beam size, although to obtain an emittance one must resolve the hyperbolic minimum. In our case, the phase shift is proportional to a longitudinal focusing strength, and the slope is proportional to the bunch length. Using Eq. 8 and Eq. 13, plotting \(x\) versus \(\phi\) will give a hyperbolic plot, the asymptotes of which will have a slope proportional to the bunch length. Fitting \([x = a^2 + b^2 \phi^2]\) to the results, the bunch length will be given by

\[
z_0 = \frac{b(E_{inj} + E_{NL})}{\eta_x E_{NL} k} \tag{14}
\]

To obtain a longitudinal emittance, one must separate the transverse contribution of the energy spread from that of the transverse emittance. Measurement of the width of the beam from minimum size to a factor of three larger is usually adequate to determine the asymptotic slope and therefore the bunch length. These same relationships apply to full bunch length (or edge emittance) when the boundary of the beam is well-defined.

**EXPERIMENTAL RESULTS**

The measurement is done by putting the bunch on crest to insure minimum energy spread. As the NL gang phase changes, the energy spread will change and the spot size will increase as shown in Fig. 1. When the NL gang phase changes in the positive direction, the beam spot shows a tail which increases with increasing phase. The dispersive contribution becomes dominant as shown in Fig. 1b - Fig. 1e. When the phase goes in the negative direction, the tail is still there but inverted as shown in Fig. 1f - Fig. 1i.

It is convenient to calibrate dispersion and beam size together using camera pixels, as shown in Fig. 2. We change the beam energy by \(\pm\) 1 MeV and measure beam displacement on the SLM (for example, if the total energy is 1000 MeV and the energy gain is changed by 1 MeV then the relative energy shift is \(1 \times 10^{-3}\)).

Three different measures are used to estimate the bunch length. The first is an edge measurement done by measuring the spot size from head to tail, this is done to overcome camera saturation and provides an upper bound for bunch length. The other estimates are rms based and suffer from camera saturation. Both RMS calculations begin by using a MATLAB code to remove the image frame, subtract the background and convert the image into a 2D matrix.
Table 1: Bunch Length Results

<table>
<thead>
<tr>
<th>Location</th>
<th>Head to tail ($\mu$m)</th>
<th>RMS ($\mu$m)</th>
<th>Stat. RMS ($\mu$m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARC1</td>
<td>474.1±20.4</td>
<td>91.4±6.5</td>
<td>86.7±2.8</td>
</tr>
<tr>
<td>ARC2</td>
<td>401.2±18.8</td>
<td>112.8±5.8</td>
<td>90.8±5.3</td>
</tr>
</tbody>
</table>

A horizontal projection is done to the output matrix for each figure. A Gaussian fit is done to the profile and the resulting sigma is taken as the beam width. The statistical RMS uses the same profile and calculates the direct rms. Both rms calculations are expected to be overestimated because of camera saturation. The three results are shown in Figs. 3, 4 and 5. The slope of Fig. 3 has the highest slope which indicates a higher bunch length, this is expected since the bunch length for Fig. 3 is a head to tail measurements. The same measurements and calculations are done for ARC2 and the results for both ARCS are shown in Table 1.

The bunch profiles for opposite polarity phase changes should approach mirror symmetry for large phase change, for which the correlated energy spread dispersively dominates the emittance driven beam size. Deviations from mirror symmetry can help identify internal beam structure.

CONCLUSION

Bunch length measurements using SLM is a fast and cost effective method at present. It is the only method in CEBAF to determine the bunch length after compression. The results show good internal agreement, especially to the statistical calculations. In the future, camera saturation will be overcome by using filters or changing the integration time of the camera.

REFERENCES