BENCHMARKING THE CERN-SPS TRANSVERSE IMPEDANCE MODEL WITH MEASURED HEADTAIL GROWTH RATES
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Abstract
The latest SPS transverse impedance model includes kicker magnets, wall impedance, transition pieces (e.g. flanges and vacuum chamber discontinuities), beam position monitors and RF cavities. The model has already been successfully benchmarked against coherent tune shift and transverse mode coupling instability measurements. In this paper we present measurements of the headtail growth rates for a wide range of negative chromaticities and for two different configurations of machine optics (nominal and low gamma transition). The measurement results are compared with HEADTAIL simulations using the wake fields obtained from the SPS transverse impedance model.

INTRODUCTION
In the framework of the SPS upgrade project an accurate impedance model is needed in order to determine its effect on the beam stability and assess the impact of the new devices to be installed in the machine, with both the present and future beam parameters [1]. The SPS impedance model is obtained by summing the contributions of the different devices along the machine (β-weighted for the transverse impedance). Analytical models, 3-D simulations and bench measurements are used to estimate these contributions. The SPS impedance model is dynamical because it needs to be updated to include newly identified impedance sources as well as modifications of installed elements or new elements.

THE LATEST SPS IMPEDANCE MODEL
The present version of the SPS transverse impedance model, which we present here, includes the following contributions:

- Kicker magnets. They are likely to be the most important impedance source in the SPS. In a very simple approximation a SPS ferrite loaded kicker can be modelled as two parallel plates of ferrite. For this simple geometrical model all the impedance terms (longitudinal, driving and detuning horizontal and vertical impedances) have been calculated analytically. CST 3D simulations were found to be in very good agreement with the analytical results. The excellent agreement between analytical model and numerical simulations can be read as an important benchmark for the simulation code in the correct solution of electromagnetic problems involving dispersive materials such a ferrite. In the framework of an improvement of the kicker impedance model we performed a step by step simulation study starting from the simplest model and introducing one by one the new features that make the model gradually closer to reality. This approach allows for a good understanding of the different contributions brought to the kicker impedance by the different aspects. First, the ferrite is assumed to be C-shaped and the whole finite length device is inserted in the vacuum tank and equipped with an inner conductor [2]. In order to further approach a more realistic model other aspects have to be included: the cell longitudinal structure, also called segmentation, which determines a significant increase of the beam coupling impedance for the SPS injection kickers (due to the short cell length) and the serigraphy for the SPS extraction kickers. All the details about the SPS kicker impedance model can be found in Ref. [3];

- Wall (resistive wall and indirect space charge), based on analytical calculation taking into account the different SPS vacuum chambers [3];

- Beam position monitors, based on CST 3D simulations [4];

- RF cavities, based on CST 3D simulations [5];

- Broadband impedance from step transitions, based on the information for the SPS flanges collected during the task force for the identification of the longitudinal impedance source responsible of the impedance peak at 1.4 GHz observed during beam measurements [6]. The broadband impedance of the SPS transitions has been calculated as:

\[
Z_{\text{transitions}} = \sum_{i=1}^{N} Z_i n_i
\]

where \(N\) is the number of different transition types, \(Z_i\) is the \(\beta\)-weighted broadband impedance of the transition \(i\) and \(n_i\) is the number of occurrences of the transition type \(i\). The broadband impedance contribution of each type of transition has been obtained by means of CST 3D EM simulations.

Figure 1 shows the full SPS impedance model including all the impedance sources analyzed weighted by the respective length and beta functions for the horizontal and vertical driving and detuning impedances [3].

BENCHMARKING THE SPS TRANSVERSE IMPEDANCE MODEL
The model has been found to reproduce with very good accuracy coherent tune shift measurements in both trans-
verse planes [7]. Moreover, macroparticle simulations using the latest SPS impedance model show a very good agreement also with TMCI instability measurements [7]. More details on the SPS impedance model and the relative impact of the different elements to the global SPS impedance can be found in Refs. [7, 8].

Headtail Growth Rates

The headtail instability growth rates are proportional to the real part of the dipolar effective impedance [9, 10]. The headtail mode \( l \) is unstable when the real part of the transverse effective dipolar impedance is negative (positive growth rates). The real part of the transverse impedance is an odd function and is defined positive for positive frequencies. Therefore, above transition energy the headtail mode \( l = 0 \) is expected to be unstable for negative chromaticity \( \xi \) (negative chromaticity frequency shift \( \omega_\xi \)). The high order modes stability depends on \( \omega_\xi \) and on the beam coupling impedance spectrum. Usually, above transition energy and for \( \omega_\xi \ll c k(\sigma_z) \) (chromaticity frequency shift much smaller than the frequency cutoff of the bunch spectrum) the high order modes result unstable for positive chromaticity and stable for negative chromaticity. In the previous condition \( k(\sigma_z) \) represents the propagation constant in vacuum for a wavelength equal to the rms bunch-length \( \sigma_z \). The stability situation is reversed below transition energy.

**SPS headtail instability growth rates**

The CERN-SPS accelerator is working above transition energy. Therefore, during standard operation the natural chromaticity is corrected to get positive values and then make headtail mode \( l = 0 \) stable. The mode is expected to be unstable for negative chromaticity with growth rates of the instability depending on the real part of the driving impedance [10]. The instability growth rates versus the chromaticity frequency shift give important information about the impedance spectrum [11, 12]. Measurements of the SPS vertical headtail instability growth rates have been performed for both the standard Q26 optics and the new low \( \gamma \) transition Q20 optics [13] (\( Q \) stands for tune and the number indicates the integer part of the transverse tune). The vertical centroid motion has been measured turn by turn for several chromaticity values by using the SPS BPM system. Several measurements have been performed at each chromaticity value. In order to obtain the growth rates, the vertical centroid motion data have been correlated to the driving impedance model, using the SPS impedance code [14], which allows calculating the growth rates for different bunch spectra.
Figure 3: Comparison between measurements and simulations of the intra-bunch motion over 21 consecutive turns for $\xi = -0.067$ (top) and $\xi = -0.967$ (bottom).

been post-processed using the procedure detailed in [10]. During measurements, bunch-lengths from the SPS mountain range and bunch intensities from the CERN-PS properly calibrated by using the SPS BCT (beam current transformer) have also been acquired [10]. More details about the measurements can be found in Ref. [10].

HEADTAIL [14] simulations have been performed by using the wake fields obtained from the SPS transverse impedance model. The simulations have been performed using the measured bunch intensities and bunch lengths and accounting for the nonlinear chromaticity (second and third order) and the double RF system. Therefore, the simulated vertical centroid motion data are post-processed as done for the measurements data to obtain the simulated growth rates. Figure 2 shows the instability growth rates as a function of the chromaticity frequency shift for Q20 and Q26 optics. The broadband behavior of the instability growth rates follows the broadband behavior of the SPS kicker impedance [3]. This similarity becomes evident for the Q26 optics where the chromaticity shift is much larger due to the smaller slippage factor. The growth rate measurements, similarly to the real part of the driving impedance of the SPS kicker impedance model exhibit a broadband behavior with a maximum around $1 - 1.1$ GHz [3, 8]. Measurements and simulations have been found in very good agreement also in the intra-bunch motion of the headtail instability [10]. The measurements data have been acquired by using the SPS headtail monitor. As example Fig. 3 shows a comparison of the simulated and measured intra-bunch motion for a chromaticity $\xi = -0.067$ and $\xi = -0.967$.

**CONCLUSIONS**

The HEADTAIL simulations using the present version of the CERN-SPS transverse impedance model have been found to reproduce the headtail vertical instability measurements (growth rates and intra-bunch motion). This result, together with the successful benchmark for the coherent tune shift of the mode $l = 0$ and for the TMCI instability behavior [7, 8, 10, 15] (intensity thresholds and intra-bunch motion) make the SPS a fine example of a machine, whose impedance model can reproduce experimental results with an unprecedented degree of accuracy for particle accelerators. Such an accurate impedance model can be used to drive machine impedance optimization and to estimate the impact on the beam stability of accelerator elements before installation in the machine. The models must be kept up to date according to modifications. Measurements of the tune shifts versus chromaticity are foreseen to further benchmark the impedance model.

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