Abstract

This paper gives a brief overview of some aspects of the beam dynamics effects induced by the natural chromaticity in the eRHIC FFAG lattice.

INTRODUCTION

A Fixed Field Alternating Gradient (FFAG) doublet-cell version of the energy recovery recirculator of the eRHIC electron-ion collider is being investigated [1, 2, 3]. A pair of such FFAG rings placed along RHIC recirculate the electron beam through a 1.322 GeV linac (ERL), from respectively 1.3 to 6.6 GeV (5 beams) and 7.9 to 21.2 GeV (11 beams), and back down to injection energy. A spreader and a combiner are placed at the linac ends for proper orbit and 6-D matching.

SIMULATION CONDITIONS

The second, 11 beam, 21.2 GeV ring is considered in this discussion for convenience. The cell is shown in Fig. 1, there are 138 such cells in each one of the 6 eRHIC arcs. The 6 long straight sections (LSS) use that very cell, with quadrupole axes aligned. In the twelve, 17-cell, dispersion suppressors (DS) the quadrupole axes slowly shift from their distance in the arc, to zero at the LSS, in a reputedly “adiabatic” manner (details can be found in [3]).

Figure 1: Arc cell in the 7.944-21.16 GeV eRHIC ERL ring [4].

Energy dependent cell tunes and chromaticities are displayed in Fig. 2. Additional details concerning the lattice and beam dynamics can be found in [5].

A complete ring is considered in the simulations discussed in the following,

\[ 6 \times \left[ \frac{1}{2} \text{LSS} - \text{DS} - \text{ARC} - \text{DS} - \frac{1}{2} \text{LSS} \right] + \text{Linac} \]

The origin of the ring (the location where the bunches are injected) is taken in the middle of the LSS since the reference orbit is zero there, whatever the energy.

SR energy loss is roughly compensated at the linac: the actual energy gain is 1.322 GeV + half the energy loss at the previous pass + half the energy loss at the next pass. In particular the starting energy is 7.944 GeV + half the energy loss at pass #1.

EMITTANCE GROWTH

Due to the large chromaticity (Fig. 2), any beam misalignment results in its phase extent in phase space, following

\[ \Delta \phi = 2\pi \xi \delta E/E. \]

SR for instance is an intrinsic cause since it introduces both energy spread and beam shift [5]. This SR effect is small however compared to nominal beam emittances, it is illustrated in Fig. 3 which shows the phase-space portrait acquired by a bunch launched with zero emittances and energy spread, after a single 11 GeV pass in the eRHIC ring.
(used in the following) of that chromaticity related effect in terms of the rms emittance, namely, surface in phase space 
\[ \varepsilon_x = 4\pi \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \] (same for \((y, y')\) space), which is thus an apparent emittance, including momentum spread induced surface increase.

Since the chromaticity is not corrected in the eRHIC linear FFAG lattice, and given the natural beam energy spread \( \sigma E / E \) in the \( 2 \times 10^{-4} \) range, thus the emittance growth is prohibitive in the absence of orbit correction. This is illustrated, for the horizontal motion, in Fig. 4 which shows the phase space portraits of a 5000-particle bunch at the end of pass 11 (21.2 GeV, collision energy), and at the end of pass 21 (back to 7.944 GeV), whereas initial conditions at start, 7.944 GeV, were Gaussian rms \( \varepsilon_x \approx \varepsilon_y \approx 50 \pi \mu m \) and \( dE / E \in [10^{-4}, +10^{-4}] \) (random uniform).

Fig. 5 summarizes the overall apparent emittance increase, over the 11 accelerated passes (from 7.944 to 21.16 GeV) followed by 10 decelerated passes (from 21.16 back to 7.944 GeV), for a bunch launched at 7.944 GeV with initial Gaussian rms \( \varepsilon_x \approx \varepsilon_y \approx 50 \pi \mu m \) and \( dE / E \in [10^{-4}, +10^{-4}] \) (random uniform). In this simulation there is no vertical orbit defect whereas the bunch is (i) experiencing small misalignments in the dispersion suppressors that cause betatron oscillations in the mm range, and (ii) recentered on the theoretical reference orbit once per eRHIC turn, at the linac (i.e., center of an LSS). Bunch distortion in phase space (similar to what is observed in Fig. 3) is at the origin of the steps (local apparent emittance increase) in the region \( a \gamma \approx 27 \) on the accelerating phase and \( a \gamma \approx 38, 28 \) on the decelerating phase.

Figure 6 shows the much reduced emittance growth in the presence of orbit control, namely here, bunch recentering at each LSS.

Figure 7 is obtained in the case of a vertical orbit defect caused by a small dipole error \( \alpha_0 \in [-1, +1] \) Gauss, random uniform, injected in all the quadrupoles of the ring. The bunch in this case is recentered at the linac, in both transverse planes, at each turn.

Figure 8 displays the evolution of the polarization (the projection, \( \cos(\Delta \phi) \), of the 5000 spins on the average spin direction) and of the spin angle spread \( \sigma \phi \), in the previous conditions of orbit defects: the polarization appears marginally sensible to misalignment effects of this nature and at this level. Note that the number of precessions \( a \gamma \),

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right vertical axis) slightly differs from an integer value, this is just an indication of a residual effect in the present rough compensation of SR energy loss.

**MULTIPLE-BEAM ORBIT CORRECTION**

A first approach to multiple-beam orbit correction uses a matching procedure, in which the theoretical FFAG orbit is imposed on the bunch centroid in the arcs, for each energy. The constraint is imposed every 23 cells, this makes 6 such sections to be corrected in a 138 cell arc. That allows 23 variables (H-correctors at quadrupoles) for 22 constraints (x and x’ for each one of the 11 energies, in one go). A 50 particle bunch is considered for the matching.

As an illustration, a strong horizontal orbit defect is injected in the arc quadrupoles, namely, a vertical dipole error $b_0 \in [-20, +20] \text{ Gauss}$ random defect.

Figure 8: Polarization, spin angle spreading, precession, in the conditions of Figs. 5, 6.

An option in this method is to apply the constraint cell after cell, in a running mode all around the ring (in both planes in addition, in the presence of both horizontal and vertical multipole defects), until the residual orbit causes tolerable residual emittance growth.

A different type of constraint, rather than the theoretical FFAG orbits, is to request minimal bunch oscillation amplitude in the cells, leaving the average orbit free. This would have the merit of allowing a self-adjustment of the FFAG orbit on the actual bunch centroid energy (which is not the design one, due to SR for instance). This is an on-going study.

**ORBIT CORRECTION, SECOND STYLE**

A second style of orbit correction in eRHIC FFAG, under prospective, is the global orbit correction. Due to the large chromaticity of the lattice, the measurable orbit (the centroid position of a bunch of particles) is deviated from the orbit of the on-momentum particle with the presence of lattice errors, as discussed earlier. Furthermore, the measurable orbit response is different from the orbit response of the on-momentum particle as well. In simulations, the simplest case that can be studied is to correct the on-momentum particle orbit using the on-momentum particle orbit response. For corrections in real machine however, one can only measure the centroid position. And ideally the measured orbit response should be used in correction. By contrast, this is not necessary in a machine where the chromaticity effect on orbit is negligible.

There are two questions associated with the global orbit correction scheme. The first one is whether correction based on on-momentum orbit and response in simulation is realistic, or whether the correction based on real orbit and response can perform as good as the simulations yield. The second question is whether it is acceptable to correct the orbit on the actual bunch centroid energy (which is not the design one), due to SR for instance. This is an on-going study.

**REFERENCES**


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