CST SIMULATIONS OF THz CHERENKOV SMITH-PURCELL RADIATION FROM CORRUGATED CAPILLARY

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Abstract
Cherenkov Smith - Purcell radiation (SPR) from a corrugated channel in infinite dielectric material is simulated and compared with a theoretical investigation for such geometry. Dependencies of Cherenkov and SPR intensities on the corrugation depth and the internal radius of the channel are discussed. A corrugated capillary with partial metal coating is also considered in the simulations in order to obtain optimal values of the corrugation depth and the external radius of the capillary.

INTRODUCTION
Terahertz frequency range includes frequencies from 0.3 to 10 THz. This part of electromagnetic spectrum has a variety of potential applications ranging from fundamental, such as studies of physical systems dynamics, to security applications, such as screening of concealed materials [1,2]. Further advances in development of a linac based, tunable and narrow band coherent source of THz radiation are very important. Mechanisms of Cherenkov radiation and SPR may be used for generation of THz radiation via coherent emission [3, 4].

In this report we discuss a hybrid mechanism for generation of coherent THz radiation based on Cherenkov and SPR, produced when a short (100 fs) electron bunch travels through a corrugated channel in dielectric material. The radiation generated by the corrugated channel was simulated using CST (Computer Simulation Technology) Particle Studio (PS) and compared with the theoretical study developed for a corrugated channel in infinite material [5]. LUCX accelerator at High Energy Accelerator Research Organisation (KEK) has been upgraded by introducing a femtosecond Ti:Sapphire laser system and is currently able to generate short, tens to hundreds femtosecond duration electron bunches [6,7]. For a proposed experimental study at LUCX facility at KEK in Japan SPR will be generated in a dielectric capillary with partial metal coating acting as a radiation reflector; the radiation will leave the capillary through the outer boundaries, not covered by the reflector. This geometry allows for generation of narrow-band, coherent SPR in THz region; as well as for more efficient radiation generation, compared, for example, to a flat diffraction grating.

Theoretical Background
Theoretical calculations of the Cherenkov Smith - Purcell radiation from a corrugated channel in infinite dielectric are based on the method of polarization current density (see chapter 4 in [8]). Coulomb field of moving electrons polarizes the material and as a result each elementary volume of the material emits radiation. Electrons in the bunch are distributed with a Gaussian distribution. The polarization currents produce secondary electromagnetic field which then propagates through the material. The spectral - angular distribution of the radiation is given in [5]. The positions of the Cherenkov peak and the SPR peaks satisfy the following dispersion relation:

$$\cos \theta = \frac{2\pi m}{kd} + \frac{1}{\beta \sqrt{\varepsilon \omega}}; \quad (1)$$

where $\theta$ is the polar angle depicted as Theta in Fig. 1; $\beta$ is the charge speed in terms of the speed of light; $k$ is the wave number in the dielectric; $d$ is the groove period; and $m$ is a diffraction order. The value of $m = 0$ corresponds to the Cherenkov peak, and the values of $m = \pm n; n = 1,2,3...$ correspond to the peaks of SPR.

SIMULATION GEOMETRY
The simulations are performed using CST PS Particle In Cell (PIC) solver [9]. The simulated geometries are shown in Fig. 1 and in Fig. 2. Figure 1 shows the geometry used for a comparison with the theory; the calculation domain is filled with dielectric in the radial direction, which in combination with open boundary conditions creates a quasi - infinite geometry. An electron bunch propagates through the channel producing Cherenkov and SPR at angles Theta satisfying the dispersion relation (1).

Figure 1: Geometry 1, quasi-infinite in r direction.

Figure 2 depicts the capillary with the reflector. The electron bunch travels with an offset in the capillary in order to generate Smith-Purcell radiation more efficiently. Both in Fig. 1 and Fig. 2 black dashed lines show non-reflective borders (open boundaries) in the calculation domain; and the red dashed lines show reflective borders. Values of the electric field are calculated at each
discretization point in the domain, and the electric field values at the outer borders of the calculation domain are extrapolated using the method described in [10] in order to obtain electric field values in the wave zone.

Figure 2: Geometry 2, a capillary with a radiation reflector.

Parameters of the two considered geometries are shown in Table 1. For geometry 2 the internal radius \( r_1 \) was chosen to be larger in order to accommodate transversely larger bunch, propagating with an offset through the capillary. The groove period and depth for geometry 2 are chosen in such a way, so that (-1) order of SPR corresponding to 300GHz is observed at \( \Theta = 89.94 \) deg.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Geom. 1</th>
<th>Geom. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lorentz-factor, ( \gamma )</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Bunch charge</td>
<td>0.1 nC</td>
<td>0.1 nC</td>
</tr>
<tr>
<td>Bunch ( \sigma_{LONG} )</td>
<td>0.1mm</td>
<td>0.03mm</td>
</tr>
<tr>
<td>Bunch ( \sigma_{TRANSY} )</td>
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<td>0.3 mm</td>
</tr>
<tr>
<td>Frequency</td>
<td>300GHz</td>
<td>300GHz</td>
</tr>
<tr>
<td>Wavelength</td>
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<td>1mm</td>
</tr>
<tr>
<td>Material</td>
<td>Fused quartz</td>
<td>Fused quartz</td>
</tr>
<tr>
<td>Dielectric permittiv., ( \varepsilon )</td>
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<td>3.75</td>
</tr>
<tr>
<td>Internal radius, ( r_1 )</td>
<td>Variable</td>
<td>2 mm</td>
</tr>
<tr>
<td>External radius, ( r_3 )</td>
<td>Infinite</td>
<td>Variable</td>
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<tr>
<td>Groove depth, ( a )</td>
<td>Variable</td>
<td>Variable</td>
</tr>
<tr>
<td>Number of periods</td>
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<td>20</td>
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<tr>
<td>Groove width, ( l )</td>
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<td>0.5 mm</td>
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<tr>
<td>Groove period, ( d=2l )</td>
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<td>1 mm</td>
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<tr>
<td>Substrate width, ( w )</td>
<td>Infinite</td>
<td>Variable</td>
</tr>
<tr>
<td>Bunch offset, ( h )</td>
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<td>1.3 mm</td>
</tr>
</tbody>
</table>

**DISCUSSION OF RESULTS**

Comparison with the Theory (a Simulation of Geometry 1)

Simulations of Cherenkov radiation intensity and SPR of (-1) order, corresponding to the dispersion relation (1), are shown in Fig. 3 and Fig. 4. The points corresponding to the CST simulation were obtained in the following way: the average value of the radiation intensity was calculated for multiple azimuthal angles Phi over the range (-180, 180) deg. for a given \( r_1 \) or \( a \) and then a standard deviation of each data set was calculated (it is depicted as an error bar for each data point). The theoretical curves were plotted using the analytical formula for the spectral-angular radiation distribution from a channel in infinite material [5, 11]; where we also reported a comparison between the theory and the simulation for the Cherenkov peak only. In this report we investigate the behaviour of the SPR peak in relation to the Cherenkov radiation peak.

In Fig. 3 the theoretical curves show emphasized oscillations, compared to the simulated curves, demonstrating more gradual radiation decrease.

Figure 3: Radiation intensity dependence on \( r_1 \) for geometry 1, \( a = 0.2 \) mm.

Figure 4 shows qualitative similarity in the behaviour between the theory and the simulation for the Cherenkov peak. There is a two-lobe intensity dependence, which is again more emphasized for the theoretical curve. For the SPR peak, however, the simulation shows gradual increase of the SPR intensity up until the value of \( a = 0.8 \) mm, while the theoretical curve shows a minimum at the value of \( a = 0.6 \) mm and the radiation intensity is considerably smaller.

Figure 4: Radiation intensity dependence on \( a \) in geometry 1, \( r_1 = 0.3 \) mm.

It is important to point out some difference between the theoretical model and the simulated geometry 1. The theoretical model was developed for a corrugated channel in infinite material, while the CST model was defined in a cubic calculation domain with open boundary conditions. Definition of the calculation domain in such a way creates irregularity in the azimuthal distribution of the radiation.
in the range of azimuthal angles $\phi (-180, 180)$ deg. Additionally, there are reflective borders in the simulation geometry, shown as red dashed lines in Fig. 1, which are not taken into account by the theory. At the same time, the theory is idealised and the oscillating behaviour of radiation intensity has to be validated, especially in the case of transversely and longitudinally Gaussian distributed bunch.

**Simulation of Geometry 2**

The main motivation for the simulation of geometry 2 is to perform an optimisation of the capillary with the radiation concentrator for the experimental study at LUCX facility. In order to propagate an electron bunch with $\sigma_{\text{TRANSV}} \approx 300\mu m$ through the capillary, its internal radius is chosen to be $2\ mm$. Electron bunch moves through the capillary along the axis $z$ with an offset (Fig. 2) in order to achieve efficient generation of SPR. It is interesting to simulate how the groove depth ($a$) and the substrate width ($w$) variations affect a spectral intensity of emitted SPR radiation. In geometry 2 the SPR is the primary mechanism of THz radiation, because at the angles $\Theta$ close to $90$ deg. the radiation is reflected by the metal coating and emitted without being internally reflected from the capillary walls.

Each point in Fig. 5 and Fig. 6 is obtained in the following way: values of the electric field components are recorded at the point $(r = r^2 \lambda; \Theta = 90$ deg.; $\phi = 90$ deg.) in the time domain, after that Fourier transform is performed in order to obtain a frequency representation of the $\Theta$ component of the electric field at this point, and then the maximum spectral intensity in the frequency range (290 - 310) GHz is taken as a data point. Fig. 5 demonstrates the dependence of the SPR spectral intensity on the corrugation depth $a$, when the substrate is absent, so we consider a set of rings enclosed by the radiation reflector. Fig. 6 demonstrates the SPR spectral intensity dependence on the substrate depth for the fixed corrugation depth $a = 0.2\ mm$.

Both Fig. 5 and Fig. 6 show dependencies close to periodic, however it is not straightforward to evaluate the period of the oscillations. Moreover, we consider this dependence for the case of a non-central propagation and with the presence of the radiation reflector, which makes it a lot more complicated for analytical calculations. The first maximum of the spectral intensity observed for the values of $a$ and $w$ equal to $0.2\ mm$. Similarly, in [12] it was shown that the spectral intensity of the $300GHz$ SPR from a lamellar grating as a function of the strip depth demonstrates oscillating behaviour with the maximum at small strip depths, corresponding to the values close to $\lambda/4 = 0.25mm$.

**CONCLUSION**

In this report we considered a hybrid mechanism for the generation of coherent THz radiation from a corrugated channel in dielectric. Comparison with the theoretical study showed qualitative agreement for the Cherenkov radiation, however some differences were identified for the SPR. The simulation of the capillary with the reflector was performed in order to optimise the capillary corrugation for the experimental study at LUCX facility at KEK. As a continuation to the presented work, modifications to the radiation reflector will be considered in order to improve a directivity pattern of the SPR. Additionally, other geometries for the radiation output will be investigated to make use of both mechanisms for THz radiation generation.

**ACKNOWLEDGMENT**

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**REFERENCES**


[9] https://www.cst.com/Products/CSTPS

