Abstract

Variable-gap undulator systems are widely used in storage rings and linear accelerators to generate soft- and hard x-ray radiation for the photon science community. For cases where the effect of undulator focusing significantly changes the electron beam optics, a compensation is needed in order to keep the optics constant in other parts of the accelerator. Since 2010, the free-electron laser (FEL) facility FLASH is equipped with two undulator sections along the same electron beamline. The first undulator is a variable-gap system used for seeding experiments, the second undulator is a fixed-gap system which serves the user facility with FEL radiation. Varying the gap in the first undulator will change the beam optics such that the FEL process in the second undulator is dramatically disturbed. For the correction of the beam optics an analytical model is used to generate feed forward tables which allows to make part of the beamline indiscernible for the subsequent sections. The method makes use of the implicit function theorem and can be used for any perturbation of the beam optics. Here, we present the method and its implementation as well as measurements performed at FLASH.

INTRODUCTION

The super-conducting linear accelerator FLASH at DESY in Hamburg serves two parallel undulator beamlines, FLASH1 and FLASH2. While FLASH2 is currently being prepared for user operation, FLASH1 has been delivering soft x-ray photons for user experiments since 2005 [1]. As shown in Fig. 1, upstream of the FLASH1 main undulator an experimental setup for FEL seeding (sFLASH) is located. It features two electro-magnetic undulators as well as four variable-gap undulator modules and thus is the ideal testbench for the study of several different seeding schemes [2].

The focusing properties of the seeding undulators in the vertical plane will have an impact on the beam optics at the entrance of the main undulator. At lower beam energies this change can easily be in the order of several tens of percent and thus significantly decreases the FEL performance in the main undulator. Therefore, the disturbance introduced by the seeding radiators has to be corrected for by adjusting the strength parameters of quadrupole magnets in that section to match the optics in the main undulator again.

While most algorithms typically find a suitable correction by numerically solving minimization problems, we present an analytical method. It is based on the implicit function theorem (compare for instance [3]) and allows for the determination of correction parameters as a function of the introduced disturbance. Furthermore, we present measurements confirming the calculations.

THE METHOD

During these calculations the optics will be described employing transverse linear beam optics. We denote the $4 \times 4$ transformation matrix describing the effect of the entire section of the beamline as $M(\rho)$, where $\rho \in \mathbb{R}^n$ is a set of all the machine parameters which are needed to fully define the optics, e.g. strength parameters of quadrupole magnets and undulator gaps. We define two machine states $\rho$ and $\rho_0$ to be equivalent if their matrices are identical

$$M(\rho) = M(\rho_0).$$

The goal of our calculations is to find the disturbed state $\rho$ that is equivalent to the undisturbed case $\rho_0$ to keep the optical functions after the section constant. For this purpose we split $\rho$ into the parameters that introduce the disturbance $\sigma$ (here, undulator gaps) and the ones that are used for the correction $\tau$. Under the assumption of uncoupled matrices, $M(\rho)$ consists of two $2 \times 2$ block matrices. Since the determinants of both block matrices have to equal unity, Eq. (1) can be reduced to 6 equations of the form

$$\Delta M_{i,j}(\rho, \rho_0) = M_{i,j}(\rho) - M_{i,j}(\rho_0) = 0.$$  

By defining a $C^1$ function $B : \mathbb{R}^n \to \mathbb{R}^6$ to have these six matrix element differences for a given, constant $\rho_0$ as its components, the set of equations (2) can be written as

$$B(\sigma, \tau) = 0.$$  

The implicit function theorem now assures the existence of a unique function $\tau(\sigma)$ around $\rho_0$, so that

$$B(\sigma, \tau(\sigma)) = 0,$$

if the Jacobian determinant $\det J_B(\tau)|_{\rho_0} \neq 0$, which we at this point will assume to be true [3]. We see, that $B$ and $\tau$ have to be of the same dimension for the correction function to be unique, meaning that the desired compensation can be achieved by adjusting six correction parameters. The derivative of this correction function can be obtained by analytical means from Eq. (4).

$$\frac{d\tau(\sigma)}{d\sigma} = -\left(\frac{\partial}{\partial \tau} B(\sigma, \tau(\sigma))\right)^{-1} \frac{\partial}{\partial \sigma} B(\sigma, \tau(\sigma)).$$

This defines a system of six coupled differential equations. Therefore, by solving this system the sought correction function $\tau(\sigma)$ can be obtained. Due to the complexity of the involved terms it will in general not be possible to find the solution analytically and a numerical approximation is needed.
A Mathematica [4] script has been written to calculate the needed matrix elements from the single transformation matrices, as well as to find the numerical solution using a simple Runge-Kutta method. The accuracy of the calculated correction functions could be verified by ELEGANT [5] simulations, as shown in Fig. 2.

Figure 1: Schematic view of the FLASH facility. The yellow boxes show the accelerator structures which serve two undulator beamlines FLASH1 and FLASH2. The seeding experiment is located upstream the FLASH1 main undulators.

The experimentally most feasible choice for the correction parameters $\tau$ are the strengths of a set of six quadrupole magnets, as they can be varied easily. Upstream of the main undulators the FLASH1 beamline features 14 quadrupole magnets available for use as correctors, which allows for 3003 different sets of 6 magnets to be selected. Our approach is to calculate the correction functions for all these combinations and select the one for which the change in the correction parameters is minimal. This minimizes undesirable side-effects, such as additional kicks introduced by the correction magnets. If the correction is to be applied continuously, additional constraints may have to be fulfilled by the functions. As the quadrupole magnets in the beamline are subject to hysteresis effects, their currents have to be varied monotonically in order for their magnetization to be well-defined, so that only monotonic correction functions are suitable for a continuous compensation.

**MEASUREMENTS**

The above described method has been tested at FLASH using the seeding undulators as a disturbance and several quadrupoles up- and downstream of that section as correction parameters. The measurements we present here have been conducted with a 1.0 GeV electron beam, closing all four seeding undulator modules and therefore maximizing the disturbance.

To verify the compensation of the disturbance, wire scanner measurements along the main undulator section of FLASH1 and with this downstream of all correction quadrupoles and disturbances have been taken. Wire scanners are able to measure the transverse size of the electron bunch and with a constant emittance along the main undulator the local beta function can be measured.

Figure 2: Development of the $\beta$-functions in dependence on the longitudinal coordinate $s$. The undisturbed state (dotted) is compared to the disturbed state with (solid line) and without (dashed) application of the correction. Grey areas in the upper plot mark the position of the variable-gap undulators, in the lower plot the quadrupole magnets used for the correction are indicated. Note that at the end of the beamline the corrected optical functions agree with the open case.

Each wire scanner has been scanned for undisturbed theory optics, the disturbed case (undulators closed) as well as the corrected case. Results are shown in Fig. 3. As can be seen from the figure, the disturbed case strongly deviates from the undisturbed theory optics, while the case where the correction was applied shows only small variance with respect to the initial case.
CONCLUSION AND OUTLOOK

An analytic approach on compensating disturbances in beam optics of particle accelerators has been developed. It allows for correcting changes in an arbitrary set of machine parameters, completely independent of the actual optical functions in the section. Because of its analytic nature, based on the implicit function theorem, it provides unique and continuous correction functions, which are suitable for a steady compensation of the disturbance. A software package allowing for easy application of these findings has been implemented.

Applying a correction will have an impact on the course of the optical functions within the compensation section. Therefore, in order to check for decisive factors (e.g. overly large β-functions), simulation codes like ELEGANT [5] still have to be employed. However, it can be assumed that the optical functions will differ less from their initial course, the more subtle the changes in correction parameters are.

A next experimental step will be to apply the correction steadily as the disturbance arises, which in the end enables us to close the seeding undulators and start seeding experiments without any notable changes of electron beam quality in the FLASH1 main undulator. This compensation constitutes an important tool for planned simultaneous operation of a seeding experiment at FLASH1.

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