BEAM-BEAM SIMULATION OF CRAB CAVITY WHITE NOISE FOR LHC UPGRADE*

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Abstract
High luminosity LHC upgrade will improve the luminosity of the current LHC operation by an order of magnitude. Crab cavity as a critical component for compensating luminosity loss from large crossing angle collision and also providing luminosity leveling for the LHC upgrade is being actively pursued. In this paper, we will report on the study of potential effects of the crab cavity white noise errors on the beam luminosity lifetime based on strong-strong beam-beam simulations.

INTRODUCTION
The High Luminosity (HL) LHC upgrade [1], aims at a tenfold increase (3000 fb⁻¹) of the integrated luminosity by 2035 as compared to its initial goal (300 fb⁻¹). This will be achieved by an increase of the instantaneous luminosity by almost an order of magnitude and therefore we expect the beam to beam electromagnetic interactions (i.e. beam-beam effects) to become stronger. It is important to evaluate the potential impact of these effects on the beam quality (e.g. emittance) in the high luminosity upgrade. In the HL-LHC upgrade, crab cavities (CCs) are proposed to compensate for geometric luminosity loss due to the crossing angle operation in collision which will lead to a 70% loss of luminosity. On the other hand, crab cavities may also have a detrimental impact on the beam quality due to imperfections. Phase noise errors in the CCs lead to a fluctuation of the beam centroid position at the interaction point, which causes emittance growth. Amplitude errors in the CCs cause bunch size fluctuations and emittance growth. Simulations were carried out to assess the implication of the phase errors for the LHC parameters [2, 3, 4]. New development in the HL-LHC design parameters and the improvement of the simulation tool to include a transverse damper model [5] demands new simulations. In this paper we present the simulation results to study the effects of crab cavity phase and voltage white noise errors on the peak luminosity of colliding beams.

COMPUTATIONAL SETUP
All simulations presented in this study were done using a strong-strong collision model implemented in the code BeamBeam3D [6]. In order to reduce numerically induced emittance growth, and to gain computation speed, the fields were computed assuming a Gaussian particle distribution, instead of a self-consistent approach. This assumption is justified by the fact that the initial Gaussian particle distribution does not change significantly in a short period of time under stable conditions. In order to keep the residual noise level low, one million macro-particles were used. The particle distribution in the longitudinal direction was divided into 8 slices. Two collisions per turn, corresponding to the interaction points (IPs) 1 and 5 in the LHC, were simulated. The crossing plane was horizontal in one IP (CMS experiment) and vertical in the other IP (ATLAS experiment). Linear transfer maps, calculated using the working point tunes, were employed to transfer the beam between collisions. The crab cavities are located 90 degrees phase advance from each IP. To model the beam transport through the crab cavity, we have assumed a thin lens approximation where the transfer map in the x-z plane is given by

\[
\begin{align*}
  x^{n+1} &= x^n + P_x^\phi + qV \sin\left(\omega z^n / c + \phi\right), \\
  z^{n+1} &= z^n + \delta E^n + qV \cos\left(\omega z^n / c + \phi\right) x^n,
\end{align*}
\]

where \(V\) is the voltage of the crab cavity, \(E_z\) is the particle energy, \(\phi\) is the phase of the cavity, and \(\omega\) is the angular frequency of the cavity. A similar transfer map with \(x\) replaced by \(y\) is used in the y-z plane.

The damper model uses a Hilbert-notch filter and two pick-ups per beam and plane, as the actual system in LHC does [7]. The correction kick at turn \(n\) due to one pick-up is given by

\[
\Delta X' = g \sum_n H_n(\phi_d) \times (X_{x,d-1,n} - X_{x,d-n}),
\]

where \(H_n\) are the coefficients of the Hilbert filter and \(\phi_d\) is the phase that needs to be determined as a function of the tune and damper gain \(g\), and \(d\) is the delay of the damper in the units of turns. The actual kick is the superposition of two terms associated with different pick-ups. In the simulation, the damper’s gain was set to 0.05 at each pickup. Noise is inserted to match the measurement [5]. The detailed physical parameters used in the simulations are given in Table 1 [8].

EFFECTS OF CRAB CAVITY PHASE WHITE NOISE ERROR

Under ideal conditions, the crab cavity will compensate the crossing angle collision completely and there is no

1: Circular and Linear Colliders
A01 - Hadron Colliders
Table 1: Physical Parameters used in the Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Np [protons]</td>
<td>$2.2 \times 10^{11}$</td>
</tr>
<tr>
<td>$\varepsilon_n$ [µm]</td>
<td>2.5</td>
</tr>
<tr>
<td>$\beta^*$ [m]</td>
<td>0.49</td>
</tr>
<tr>
<td>$Q_x$</td>
<td>62.31</td>
</tr>
<tr>
<td>$Q_y$</td>
<td>60.32</td>
</tr>
<tr>
<td>$Q_z$</td>
<td>0.0019</td>
</tr>
<tr>
<td>$\theta$ [mrad]</td>
<td>0.59</td>
</tr>
<tr>
<td>$g_1$, $g_2$</td>
<td>0.05</td>
</tr>
<tr>
<td>Damper noise</td>
<td>on</td>
</tr>
<tr>
<td>Crab cavities</td>
<td>on</td>
</tr>
<tr>
<td>Collisions [1/turn]</td>
<td>1 hor., 1 ver.</td>
</tr>
</tbody>
</table>

centroid offset between two beams at the collision point. In practice, the noise in the RF control system results in phase and amplitude fluctuation of the crab cavity field. Under a short bunch approximation, the phase error results in the centroid offset of two beams at the collision point given by the following equation:

$$
\Delta X = \frac{c}{\omega_{cc}} \tan \left( \frac{\theta}{2} \right) \delta \phi,
$$

(3)

where $\omega_{cc}$ is the crab cavity angular frequency, $\theta$ is the crossing angle, $c$ is the speed of light in vacuum, and $\delta \phi$ is the crab cavity phase error. Assuming the phase error as a white noise, the above centroid offset oscillation while colliding will result in emittance growth and luminosity degradation [9]. An analytical estimate of the luminosity degradation is given by [10]:

$$
\frac{\Delta L}{L} = 10 \left( \frac{\xi_{tot}}{\sigma} \frac{\Delta X}{\sigma} \right)^2
$$

(4)

where $\xi_{tot}$ is the total beam-beam parameter, $\Delta X$ is the stable amplitude of centroid fluctuation, $\sigma$ is the rms beam size at the IP. The amplitude of centroid fluctuation can be related to the offset caused by the random phase error for two IPs as:

$$
\Delta X = \frac{\delta X}{\sqrt{2}}
$$

(5)

where $\delta X$ is the amplitude of the crab cavity phase noise induced offset, and $g$ is the gain of the damper. Figure 1 shows the luminosity degradation rate as a function of normalized offset amplitude from the strong-strong simulation and that from the analytical model. It is seen that simulation results agree with the analytical model.

In order to maintain a luminosity lifetime on the order of 20 hours, the luminosity degradation rate needs to be kept below a level of a few percentages per hour. This suggests that the amplitude of the offset needs to be kept within a few nanometers. For the 400 MHz crab cavity used in the LHC upgrade with 0.59 mrad crossing angle, this corresponds to a few $10^{-5}$ radians of acceptable phase noise amplitude. In the above simulation, we have used a linear short bunch approximation for the phase error. Such an approximation has an advantage to connect the simulation results with the analytical model estimate. In simulation, this error can also be directly included through the Eq. 1 without the linear approximation. Figure 2 shows the luminosity degradation rate as a function of phase error amplitude using both the linear short bunch approximation model and the direct nonlinear model. It is seen that both models agree with each other very well.

Both simulation results show that the white noise phase error amplitude has to be kept within a few times of $10^{-5}$ radian in order to have luminosity lifetime of ~20 hours.

In the above study, we have assumed that a $\beta^*$ leveling scheme was used for the given event pile-up limit so that the peak luminosity is limited below $2.6 \times 10^{35}$ cm$^{-2}$s$^{-1}$ [8]. Besides using the $\beta^*$ as a method of leveling, varying crab cavity voltage can also be used as another potential option of leveling. Starting with an off-design voltage so that the crossing angle of the colliding beams is not fully compensated, this results in a lower peak luminosity below the pile-up limit. Gradually increasing the crab cavity voltage to improve the compensation as the number of protons decreases helps maintain a constant level of peak luminosity and improve the integrated luminosity. This scenario is not the baseline scenario for HL-LHC. (Actually the experiments do not like it since it increases the luminous region.) In this case, we assumed that the initial crab cavity voltage is about 10% of the full compensation voltage. Figure 3 shows the luminosity...
degradation rate as a function of crab cavity phase white noise amplitude in the case of crab cavity voltage leveling. It is seen that in this case, the tolerance for the phase error amplitude can be a few times $10^{-4}$ radians in order to maintain a good luminosity lifetime.

**EFFECTS OF CRAB CAVITY VOLTAGE WHITE NOISE ERROR**

![Figure 3: Luminosity degradation rate as a function of phase error amplitude with crab cavity voltage leveling.](image)

Figure 3 shows the luminosity degradation rate as a function of phase error amplitude with crab cavity voltage leveling.

![Figure 4: Luminosity degradation rate as a function of relative voltage error amplitude in the case of the $\beta^*$ level.](image)

Besides the phase error inside the crab cavity, there is also voltage error in the crab cavity due to RF power fluctuation. While the crab cavity phase error causes the beam centroid offset at the interaction point, the voltage error causes beam size fluctuation at the interaction point. Figure 4 shows the luminosity degradation rate as a function of relative voltage error amplitude in the case of $\beta^*$ leveling. It is seen that in order to keep the luminosity degradation rate within a few percentages per hour, the relative white noise voltage error amplitude needs to be controlled within a few times $10^{-5}$. The degradation rate goes up with the increase of the voltage error amplitude and shows a linear dependence in log scale. We also studied the effects voltage error in the case of the crab cavity voltage levelling. Figure 5 shows the luminosity degradation rate as a function of relative voltage error amplitude in the case of the crab cavity voltage levelling. In this case, the relative voltage error amplitude can be a few times $10^{-4}$ in order to keep the luminosity degradation rate within a few percentages per hour and to maintain a good luminosity lifetime.

**CONCLUSIONS**

Using a strong-strong beam-beam simulation model, in this paper, we studied the effects of crab cavity phase error and voltage error on peak luminosity degradation in the HL-LHC. Here, we investigated two scenarios: the baseline one with the $\beta^*$ leveling, and an alternative one with the crab cavity voltage leveling. We found that in the $\beta^*$ leveling case, with the current feedback control model and parameters, the phase error amplitude needs to controlled within a few times $10^{-5}$ radians and the relative voltage error amplitude within a few times $10^{-5}$ in order to maintain a good luminosity life time. In the crab cavity voltage levelling case, the phase error amplitude needs to controlled within a few times $10^{-4}$ radians and the relative voltage error amplitude within a few times $10^{-4}$ in order to maintain a luminosity life time of 20 hours. In this study, we assumed a white noise for both phase and voltage errors. These simulation results could be on the pessimistic side since the real error has a spectral power density distribution. In the future study, we will report on the results using the detailed noise spectrum in the simulations to determine the tolerance level of the noise.

**REFERENCES**
