VIRTUAL CAVITY PROBE GENERATION USING CALIBRATED FORWARD AND REFLECTED SIGNALS

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Abstract

The European X-ray free electron laser requires a high-precision control of accelerating fields to ensure a stable photon generation. Its low level radio frequency system, based on the MicroTCA.4 standard, detects the probe, forward and reflected signals for each cavity. While the probe signal is used to control the accelerating fields, a combination of the forward and reflected signals can be used to compute a virtual probe, whose accuracy is comparable to the directly sampled probe. This requires the removal of cross-coupling effects between the forward and reflected signals. This paper presents the precise generation of a virtual probe using an extended method of least squares. The virtual probe can then be used for precise field control in case the probe signal is missing or corrupted. It can also be used to detect any deviation from the nominal probe profile.

INTRODUCTION

The Free Electron LASer (FLASH) at the "Deutsches Elektronen Synchrotron" in Hamburg is a facility for research with tunable laser light. It provides its users a pulsed light in the X-ray range with tunable wavelength down to 4.2 nm generated by SASE processes. Electron bunch trains of variable length and frequency with a repetition rate of 10 Hz are accelerated to about 1.2 GeV. Each pulse is enabled for about 1.4 ms, meanwhile up to 2400 bunches with a maximum repetition rate of 3 MHz are injected. In order to provide stable and reproducible photon pulses a precise acceleration field control is needed. During the last years, several control strategies for vector-sum regulation, i.e. the sum of up to 16 cavities and its RF field probes, were developed and included in the Low-Level RF (LLRF) controller. Hereby learning feedforward (LFF) minimizes repetitive amplitude and phase errors from pulse to pulse [1], whereas the multiple input multiple output (MIMO) controller acts within the pulse [2]. The necessary RF field regulation requirements are reached and below a relative amplitude error of 0.01 % and an absolute phase error of 0.01 degree. Besides the detection of the cavity probe signal, the forward and reflected signals of each cavity at the waveguide distribution is measured, as depicted in Fig. 1. In this paper, it is shown that the latter can be used to generate a virtual probe signal usable for system health detection and failure classification. If the real probe detection fails, the virtual probe can still be used to drive the system and ensure the amplitude and phase regulation requirements.

THEORETICAL APPROACH

The main goal of this contribution is to calibrate the measured (index m) complex forward \( V_{F,m} \) and complex reflected \( V_{R,m} \) signals to the calibrated (index c) \( V_{F,c} \) and \( V_{R,c} \), respectively. An example for signal detection is shown in Fig. 2. As can be seen, the forward signal shows an amplitude value which is non-zero during decay, although the RF drive is switched off. A measurement calibration can overcome the imperfection of the signal detection, mainly caused by signal couplings at the pick-up, also visible in the reflected signal.
Introduction

The measurements for a single pulse are collected as complex value $V_{P,m}(k) = V_{P,m}(k) + i V_{P,m}(k)$ with discrete time instance $k \in [1, T]$ containing the entire RF pulse by

$$
\tilde{V}_{P,m} = \left( V_{P,m}(1) \ldots V_{P,m}(k) \ldots V_{P,m}(T) \right)^T,
$$
$$
\tilde{V}_{F,m} = \left( V_{F,m}(1) \ldots V_{F,m}(k) \ldots V_{F,m}(T) \right)^T,
$$
$$
\tilde{V}_{R,m} = \left( V_{R,m}(1) \ldots V_{R,m}(k) \ldots V_{R,m}(T) \right)^T.
$$

The calibration method assumes, that the measured probe signal is the sum of forward and reflected signal given as

$$
\tilde{V}_{P,m} = x \tilde{V}_{F,m} + y \tilde{V}_{R,m} , \quad (1)
$$

with constant complex calibration parameters $x \in \mathbb{C}$ and $y \in \mathbb{C}$, [3]. It is assumed that the measured probe signal $\tilde{V}_{P,m}$ is perfectly detected, hence constitutes a reference signal for signal calibration. Furthermore, we will assume non-zero cross-couplings between the forward and reflected signals which can be represented as

$$
\begin{align*}
\tilde{V}_{F,c} &= a \tilde{V}_{F,m} + b \tilde{V}_{R,m}, \\
\tilde{V}_{R,c} &= c \tilde{V}_{F,m} + d \tilde{V}_{R,m} , \quad (2)
\end{align*}
$$

where $a$, $b$, $c$ and $d$ are the four complex parameters to be estimated leading to the virtual probe signal

$$
\tilde{V}_{P,v} = \tilde{V}_{F,c} + \tilde{V}_{R,c} . \quad (3)
$$

Estimation Using Method of Least Squares

The estimation of $x = a + c$ and $y = b + d$ is unique within an usual RF pulse, i.e. as long as the signal shape differs for forward, reflected and probe signals. This can be checked by the rank or the singular value decomposition (SVD) of measurement matrix containing the forward and reflected signals. If the rank is two, the estimation of $x$ and $y$ is unique, while the estimation of the extended set of parameters to estimate is unique if and only if the rank is four. However it will be shown that an extended set of equations based on auxiliary conditions can be used to solve the problem.

Hereby the measurement matrix to be calibrated is given by forward and reflected signals as

$$
\begin{bmatrix}
\tilde{V}_{F,m} & \tilde{V}_{R,m} \end{bmatrix} , \quad (4)
$$

which is to be calibrated with respect to the probe signal using a method of least square (MLS) to solve $x$ and $y$ by

$$
\begin{bmatrix}
\tilde{V}_{P,m} \\
\end{bmatrix} = \begin{bmatrix}
\tilde{V}_{F,m} & \tilde{V}_{R,m} \end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} \quad (5)
$$

$$
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
\tilde{V}_{F,m}^T \tilde{V}_{R,m} \end{bmatrix}^{-1} \begin{bmatrix}
\tilde{V}_{F,m}^T \tilde{V}_{P,m} \\
y
\end{bmatrix} \quad (6)
$$

This solution is unique if and only if the rank of $A_m$ is two. This can also be checked by the singular values of SVD for the matrix $A_m$.

Estimation Using Extended MLS

In order to identify the four independent parameters, it is necessary to extend (1) to the problem given in (2). Furthermore, it is necessary to add additional constraints to cope with the rank of two limitation for the measurement matrix. First, let us consider only the decay phase to separate the entire RF pulse and the decay phase an additional superscript is introduced. The variables with superscript $\delta$ contains only the RF signals during the decay phase. During this time, the decay phase for the RF pulse is switched off, hence to the calibrated forward signal $\tilde{V}_{F,c}^\delta = 0$. Nevertheless, the RF gate may still be opened and hence a forward signal with small contribution may be observed. Furthermore, the reflected signal during decay contains the overall power from the cavity. The following equations are valid only during decay:

$$
\begin{align*}
\tilde{V}_{P,m}^\delta &= c \cdot \tilde{V}_{F,m}^\delta + d \cdot \tilde{V}_{R,m}^\delta , \quad (7) \\
\tilde{V}_{F,c}^\delta &= a \cdot \tilde{V}_{F,m}^\delta + b \cdot \tilde{V}_{R,m}^\delta , \quad (8)
\end{align*}
$$

Example ACC1 - Cavity 1: Given the decay phase of probe $\tilde{V}_{P,m}^\delta$, forward $\tilde{V}_{F,m}^\delta$, and reflected $\tilde{V}_{R,m}^\delta$ signal. The measurement matrix $\begin{bmatrix}
\tilde{V}_{F,m}^\delta & \tilde{V}_{R,m}^\delta \end{bmatrix}$ has a rank of two and singular values of $\sigma_1 \approx 522$ and $\sigma_2 \approx 0.23$, hence the solution of (7) is unique and gives $c = 0.96 + i \cdot 0.1$ and $d = 1 - i \cdot 0.05$. However such huge cross-coupling from forward to reflected signal is unlikely, hence the resulting cross-coupling parameter $c$ is ill-conditioned and needs to be constraint.

Weighting factors Two additional weighting factors are introduced to keep the order of occurring cross-couplings between the forward and reflected signal and vice versa in right dimension. Hereby the focus is on penalizing the magnitudes $|b|$ and $|c|$ to a reasonable number without loosing precision of parameter estimation. Such penalty methods, e.g. logarithmic barrier function in interior point method, are widely used in a class of algorithms to solve constrained optimization problems.

The ratio of forward to reflected signal during decay leads to an estimation of the cross-couplings by

$$
\tilde{V}_{F,m}^\delta = S_{ab} \cdot \tilde{V}_{R,m}^\delta , \quad (9)
$$

with $S_{ab} \ll 1$ and used in the following to extend the MLS.
Therefore the extended form of method of least square (with neglected measurement indication \( m \)) to solve \( a, b, c \) and \( d \in \mathbb{C} \) is given by

\[
\begin{pmatrix}
\hat{V}_P \\
\hat{V}_P^0 \\
\hat{V}_F \\
\hat{V}_F^0
\end{pmatrix} = 
\begin{pmatrix}
\hat{V}_P & \hat{V}_R & \hat{V}_F & \hat{V}_R \\
0 & 0 & \hat{V}_F^0 & \hat{V}_R^0 \\
\hat{V}_F^0 & \hat{V}_R^0 & 0 & 0 \\
0 & 0 & W_b^{-1} & 0
\end{pmatrix}
\begin{pmatrix}
a \\
b \\
c \\
d
\end{pmatrix}
\tag{10}
\]

where \( W_b = |\mathbf{S}_{ab}| \) and \( W_c = k_{\text{add}} \cdot |\mathbf{S}_{ab}| \) are weighting factors to the parameters \( b \) and \( c \), respectively. An additional variable \( k_{\text{add}} \) is introduced which is equal to one as a first assumption. We will discuss in the following section why it is necessary to tune \( W_c \) by an additional parameter.

![Figure 3: Example of uncalibrated (dashed lines) and calibrated (solid lines) signals for cavity 1 in ACC1 at FLASH with zoom in time range for both probe signals.](image)

**DISCUSSION**

Figure 3 shows an example for signal calibration for the first cavity at FLASH. There are two single spikes visible in the amplitude of the virtual probe signal \( \hat{V}_{P,v} \). On the one hand, during the transition from filling to flattop and on the other from flattop to decay. This is explainable by the hard transition during the different operation points. Furthermore, the signals are low-pass filtered from 81 MHz to 9 MHz after the ADC detection. Such filtering may lead to different sensor dynamics, while a perfect virtual probe generation requires the same dynamics for forward and reflected signal. However, such a transition can be filtered out or removed from the dataset if the virtual probe is used for driving the system. Furthermore, some of the signal calibrations may not be perfect. This can be shown by consideration of independent signals, e.g. signals not used for signal calibration, like the detuning and the half bandwidth within a pulse [4], an example is shown in Fig. 4. Especially for larger coupling coefficients, the latter are showing steps in the transition from filling to flattop and from flattop to decay which is unlikely for SRF cavities. However this information can be used to optimize the parameter estimation by an adaptive estimation process optimizing the independently generated signals (\( \Delta f \) and \( f_{1/2} \)). To do so, the additional tuning parameter \( k_{\text{add}} \) is introduced, highly suitable to weight the set of parameters to their right order.

![Figure 4: Computation of half bandwidth for calibrated signals. The computed value (black line) and filtered value (red dashed line) are shown for the entire pulse. Furthermore, the bandwidth computed from decay (single value from probe decay) is visualized for the whole RF pulse (yellow line).](image)

**CONCLUSION**

This paper describes a suitable calibration tool for forward and reflected waveguide signals. Both are calibrated with respect to the measured probe signal. The missing information about cross-couplings is solved by using an extended set of equations solved by linear regression. Additional cross-checks using independently generated signals further helps tuning the complex calibration constants to a reasonable number. Investigations considering effects from neighboring cavities, shown in Fig. 1, are in progress.

**REFERENCES**


