Abstract

The potential upgrade of the Relativistic Heavy Ion Collider (RHIC) to an electron ion collider (eRHIC) involves numerous extensive changes to the existing collider complex. The expected very high luminosity is planned to be achieved at eRHIC with the help of squeezing the beta function of the hadron ring at the IP to a few cm, causing a large rise of the natural chromaticities and thus bringing with it challenges for the beam long term stability (Dynamic aperture). We present our effort to expand the DA by carefully tuning the nonlinear magnets thus controlling the size of the footprints in tune space and all lower order resonance driving terms. We show a reasonably large DA through particle tracking over millions of turns of beam revolution.

INTRODUCTION

The potential upgrade of the Relativistic Heavy Ion Collider (RHIC) to an electron ion collider (eRHIC) [1] involves numerous extensive changes to the existing collider complex. A high intensity electron energy recovery linac (ERL) will be added to the existing RHIC facility to collide with the strongly cooled hadron beams. The expected very high luminosity will be achieved with the help of squeezing the beta function of the hadron ring at the interaction points (IP) by at least 10-fold to a few cm, from the existing RHIC operating lattices. This will cause a large rise of the chromaticities and potentially undermines the beam’s long term stability (Dynamic aperture).

It is well know that modern storage rings (both hadron rings and lepton rings) employ nonlinear magnets (sex-tupoles, octupoles, etc.) to correct the chromaticities from negative to small positive numbers to avoid microwave instabilities to develop. The natural chromaticities in the storage ring, i.e., the chromaticities rising from pure linear magnets (dipoles, quadrupoles), can be expressed as

\[ C_x = -\frac{1}{4\pi} \int \beta_x(s)K_x(s)ds, \]

and

\[ C_y = -\frac{1}{4\pi} \int \beta_y(s)K_y(s)ds. \]

Furthermore, the chromaticities rising from the IRs can be conveniently expressed as

\[ C_{IR} = -\frac{2\Delta s}{4\pi\beta^*} \approx -\frac{1}{2\pi} \sqrt{\frac{\beta_{max}}{\beta^*}}, \]

where \( \Delta s \) is the distance from the IP to the first focusing magnet. Thus by squeezing the \( \beta^* \) 10-fold, the \( \beta_{max} \) increases 10 times and \( C_{IR} \) gains a 10 times of growth. This can be the dominant contribution to the total natural chromaticities in strongly focused colliders. For eRHIC, the \( \beta^* \) for hadron lattice is 5 cm (down from 65 cm for RHIC), which results in \( C_{IR} \approx 50 \), about 2 times of the chromaticities in all the ARCs. Thus the total chromaticities \( C_{tot} \) become about 70-80 for one IP and 120-140 for two IPs.

We employ strong sextupole families to correct such high chromaticities. A schematic in Fig. 1 shows the layout of eRHIC sextupole families as the rearrangements of the existing RHIC layout for higher energy and new IR designs. At the mean time, the buses of sextupole power supplies are proposed to be rewired to form more families (24 families) of independent knobs for chromatic terms corrections (detailed in following sections).

DYNAMIC APERTURE OPTIMIZATION

As mentioned above, the sextupole strengths are strong (about 2 times of the running RHIC’s setup) for eRHIC due to the high chromaticities rising from strongly focused IRs. This generates strong chromatic aberration and high resonance driving terms (RDTs). Resonance driving terms, as a direct product from normal forming the one turn map of the storage ring whose schematic drawing can be found in Fig. 2, has an indirect impact on the particle’s long term stability know as dynamic aperture (DA).

The direct relation between RDTs and DA is not known since the particle revolution in phase space and x-y real space in existence of nonlinear elements is highly chaotic.
Figure 2: A schematic drawing showing the process of normal forming the one turn map of a storage ring, where an action-angle dependant one turn map gets transformed into an action only dependence map.

However, it is believed that lowering the RDTs, together with reducing the tune shift with amplitudes and second order chromaticities, help to increase the stable region of particle revolution, i.e., enlarge the DA size.

The RDT coefficients $h_{jklm}$ for N sextupoles, representing the lowest order RDTs, is usually calculated as

$$h_{jklm} = c \sum_{i=1}^{N} S_2 \beta_x^{(j+k)/2} \beta_y^{(l+m)/2} e^{i[(j-k)\mu_x + (l-m)\mu_y]},$$

where $S_2$ is the integrated sextupole strength. To minimize the RDTs, we need to carefully pick the sextupole strengths and locations.

In eRHIC IR design, we modified the existing RHIC’s antisymmetric IR lattice to a symmetric lattice (Fig. 3), i.e., the dispersion functions on the two sides of IPs are symmetric. Furthermore, we design the lattice to have 90 degree phase advance from IP to the first sextupole. In such a way, the chromatic contribution from sextupoles in IR (to the lowest order) is cancelled and thus not contributed to the rest of the storage ring. The ARCs in eRHIC hadron lattice, which is composed of pure FODO cells, have 90 degree phase advance per cell to remove the coupling between sextupole families. We know the 2nd order chromaticities can be expressed as

$$C_x^{(2)} = -C_x^{(1)} - \frac{J_{p,x}^2}{4(\nu_x - p/2)\delta^2},$$

where $J_{p,x}$ are the stopband integrals of resonances. The change in the stopband integrals thus reads

$$\Delta J_{p,x} = \frac{\delta}{2\pi} N[\beta_F(\Delta S)_F D_F + \beta_D(\Delta S)_D D_D e^{i\pi/4}].$$

The stopband integral of resonances (and the 2nd order chromaticities) therefore can be easily minimized with careful tuning of the 24 families of sextupoles.

**SIMULATION SETUP AND RESULTS**

We employed multiple simulation tools to study this entire process: the linear lattice design is done in SYNCH [2] and MADX [3] for their fast convergence; the optimization of sextupole setups and locations is done in ELEGANT [4]; the tracking of DA over millions of beam revolution (due to the slow cooling rate for hadron rings) is done in SimTrack [5]. A self-developed Python script is developed for conversion and gluing between programs. The beam parameters we used for the self consistent simulation are listed in Table. 1.

Figure 4 shows the DA (in terms of rms beam size) versus momentum deviation using only two families of sextupoles. The DA size quickly reduces and diminishes for particles with large momentum offsets.
With optimizing and tuning the 24 families of sextupoles as mentioned above, we achieved a much larger DA size shown as the red curve in Fig. 5. We studied the robustness of such scheme by assigning magnets misalignment (with FW 100 µm), magnets gradient errors (±0.1% for quadrupoles and sextupoles) and 6D beam-beam interactions (with beam parameter at 0.015). The results are plotted as green and blue curves in Fig. 5. The DA size larger than 20 sigmas under the worst scenarios. Our study shows pretty good robustness for such sextupole working point.

The chromaticities (Fig. 6) and chromatic aberration on beta functions (Fig. 7) are well under controlled with such sextupole setup.

Figure 5: The DA with 24 families of sextupoles has been greatly improved. The DAs for bare lattice (red), lattice with beam beam (0.015) and misalignment (FW 100 µm, green) and lattice with beam beam and misalignment and magnet gradient errors (±0.1%, blue) are shown as a result of careful optimizing and tuning of the sextupoles.

Figure 6: The chromaticities (1st and 2nd order) are well controlled after the optimization of sextupole families shown in the tune energy deviation plot.

Figure 7: The chosen setup for sextupole families also minimize the beta function at collision dependence on energy deviation.

Table 1: Parameters for eRHIC DA Optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Hadron energy (GeV)</td>
<td>100</td>
</tr>
<tr>
<td>Hadron beam emittance (µm)</td>
<td>0.2</td>
</tr>
<tr>
<td>( \beta^* ) at IP (cm)</td>
<td>5</td>
</tr>
<tr>
<td>( \nu_x, \nu_y )</td>
<td>0.82, 0.42</td>
</tr>
<tr>
<td>Maximum gradient sextupole (( S_2, 1/m^2 ))</td>
<td>1.03</td>
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REFERENCES