Abstract

We present plans for a nonlinear lattice at the University of Maryland Electron Ring (UMER). Theory predicts that a strong nonlinear lattice can limit resonant behavior without reducing dynamic aperture if the nonlinear fields preserve integrability or quasi-integrability. We discuss plans for a quasi-integrable octupole lattice, based on the work of Danilov and Nagaitsev. We use Elegant and the WARP PIC code to estimate the octupole-induced tune spread. We discuss improvements to the ring in support of octupole lattice experiments, including generation and detection of emittance-dominated, negligible space charge beams.

INTRODUCTION

Conventional accelerators are based on Courant and Snyder’s theory of the alternating-gradient synchrotron, developed in 1952 [1]. In an AG focusing lattice, particles oscillate transversely with a characteristic tune. Unfortunately, this system is susceptible to coherent resonances which lead to beam loss and halo growth. Thin lens octupoles for Landau damping limit coherent resonances, but introduce unbounded chaotic orbits leading to particle losses and limiting achievable beam intensity.

A novel approach by Danilov and Nagaitsev suggests an integrable but highly nonlinear lattice as an alternative to the standard AG synchrotron [2]. In a nonlinear lattice with a large amplitude-dependent tunes spread, particles that are resonantly driven to higher amplitudes will decohere from that resonance and have only limited amplitude growth. Effectively, the beam is immune to coherent resonances. This nonlinear detuning is also shown to mitigate halo growth driven by beam envelope mismatch [3].

The fully integrable elliptic potential will be tested at Fermilab’s IOTA ring [4]. Danilov and Nagaitsev also consider the case in which the nonlinear field is purely octupolar.

NONLINEAR UMER LATTICE

The theory of quasi-integrable nonlinear optics assumes a symmetric beam is propagating in nonlinear fields that are scaled longitudinally with the inverse of the envelope function [2]. External focusing must be provided for a bounded envelope function. To achieve this in a ring, a nonlinear channel will installed over a symmetric beam waist, and the remaining linear lattice has a transfer function equivalent to a thin x-y symmetric focusing lens. This is shown schematically in Fig. 1.

UMER is a 10 keV, 11.52 meter storage ring that supports electron beams with variable tune depressions ($\nu / \nu_0 = 0.85 – 0.15$) for the study of space charge dominated beam dynamics. The ring lattice is 36 FODO cells composed of printed circuit quadrupoles and dipoles. To modify UMER as a nonlinear optics test bed, we will superimpose an octupole channel onto the existing lattice, as shown in Figure 2.

Particles contained in an octupole potential that scales as $V(s) \propto 1/\beta_3^3(s)$ will conserve a single invariant of transverse motion, the normalized Hamiltonian, and particle orbits will be chaotic but bounded. The nonlinear optics program at UMER will test this quasi-integrable lattice by incorporating octupole magnets into the existing ring framework.

This paper describes the progress being made towards the UMER octupole lattice design and experiments. The simulation effort is focused on identifying an operating point for the lattice that maximizes beam tune spread with achievable octupole fields. Experimental work includes demonstration of a negligible space charge beam for future lattice studies.
The maximum achievable tune spread depends on the phase advance through the nonlinear channel and the octupole strength.

A useful technique for exploring the acceptance and tune spread in an octupole lattice is frequency map analysis (FMA), developed by Laskar for analysis of planetary motion [6]. This technique uses the change in frequency over many turns of a particle’s orbit, $\Delta \nu$, as a metric for nonlinear behaviour. This technique (applied in WARP [7] and Elegant [8] codes) has been compared to similar analysis for the IOTA ring (applied with the Lifetrac code [4]) to fairly good agreement.

Figure 3 shows the frequency map for a ‘toy model’ of the UMER octupole lattice. 10 keV electrons pass through an ideal octupole channel and the linear part of the lattice is condensed into a thin, 0-phase advance, X-Y symmetric focusing matrix. This map is generated with single particle tracking through an ideal lattice in the WARP PIC code [7] with no space charge. The fractional tune of the lattice is relatively low ($\nu = 0.087$) and maximum possible tune spread is predicted to be 0.10 in WARP and 0.06 in Elegant.

The FMA plots show that the regions of large octupole-induced tune shifts are close to the limit of acceptance, making it ideal to operate with a dynamic aperture approximately equal to the beam size. Figure 4 shows the aperture and tune spread as a function of fractional lattice tune and integrated octupole strength. The plateau on Fig. 4(a) is an artifact of the simulation size ($7\sigma$’s). Without this artificial limit, aperture increases with decreased field strength, until limited by the beam pipe.

It is apparent that, while the octupole tune shift increases with octupole strength, the dynamic aperture decreases. The maximum octupole strength also increases with phase advance in the channel, as max $(K_3(s) \propto 1/\beta(s)^3)$ increases. The vertical marker in Fig. 4 indicates the best operating point for the parameter space that was investigated. At a fractional tune of $\nu = 0.12$ and integrated octupole gradient of $15 \ T/\text{m}^2$, the absolute maximum tune spread is 0.08. This octupole strength corresponds with a peak strength of $54 \ T/\text{m}^3$, which is easily attainable for printed circuit octupoles, as explained below.

Based on the shape of the parameter landscape, it is desirable to push towards higher tunes and larger octupole gradients for larger tune spreads, ideally $>0.10$. 

**LATTICE TUNE**

The surrounding quadrupole lattice will be modified to provide a symmetric waist in the octupole channel by independently varying the strengths of the quadrupoles in the ring. A linear lattice solution for the nonlinear UMER ring can be found in [5], although this solution is not optimized for large tune spread.
Phase Advance in UMER

The maximum phase advance of the octupole channel will be limited by the focusing strength of the linear lattice. 0-current envelope tracking in Elegant demonstrates how the UMER lattice can be modified to imitate an nπ-phase advance focusing lens [5]. Phase advance of 0.087\(\cdot\)2π has been demonstrated using 14 quadrupoles to match a 32 cm octupole channel with a FODO lattice. Greater values are possible, but the maximum phase advance in the channel is limited by the allowable excursion of the envelope (≈ 1 meter).

Solenoids are considered for the matching section, to increase the attainable phase advance. With solenoids, phase advance of 0.4\(\cdot\)2π was obtained in a 32 cm drift. While the increase in phase advance (and therefore tune spread) is dramatic, use of solenoids creates additional alignment challenges.

OCTUPOLE DESIGN

![Figure 5: Preliminary single-layer printed circuit octupole design in Maxwell 3D.](image)

A single-layer 9-turn mock-up of a printed circuit octupole was simulated with Maxwell 3D (see Fig. 5). For 27 Amp-turns this magnet has a gradient of 140 \(T/m^3\). Much higher gradients are possible through a combination of water-cooling (up to 12 Amp excitation) and multi-layer coils.

NEGLIGIBLE SPACE CHARGE BEAM

A beam with less space charge than the existing lower-limit UMER beam (0.6 mA, \(\nu / \nu_0 = 0.85\)) is desired for nonlinear dynamics experiments, in order to see the effects of the octupole-induced tune shift without a large space charge tune shift.

UMER beams are produced by a pulsed cathode grid and apertured downstream of the source for control of the space charge concentration [9]. A nominally 50\(\mu\)A beam was produced by reducing the cathode grid bias to allow leakage current and longitudinally gating the DC electron beam with the pulsed injector dipole. This resulted in a high-emittance, low current beam that maintained a DC signal for over 1000 turns, ultimately limited by the pulse length of the injection dipole. A low current, low emittance beam produced through photo-emission with a laser pulse is currently being explored as an additional UMER operating mode.

CONCLUSION

Once the maximum achievable lattice tune is identified (given limitations of the envelope excursion and octupole strength), a new lattice solution can be found and demonstrated on UMER. This will be done simultaneously with ring improvement and characterization studies and optimization of the octupole design. Demonstration of new operating modes for low-current beams will allow decoupling of the external nonlinearities from space charge tune spreads in future octupole lattice experiments.

With all the pieces in place, UMER will be able to address the following questions: Can electrons be stably transported in a lattice with strong octupole fields? Using a low-current, high emittance beam will allow us to quantify the dynamic aperture over hundreds to thousands of turns. Are observed tune spreads equal to predicted spreads? This is best achieved with a low-current, low emittance beam. Improving BPM SNR is crucial for accurate tracking of this beam. Does halo damping occur as predicted? Halo in UMER can easily be produced through injection mismatch, as well as through excitation of the envelope modes with an electric quadrupole.

REFERENCES